You should know about exponents and logarithms from algebra. In particular, if b is a positive number and r is any real number, then you should know about b^r; if also $b \neq 1$ and t is a positive number, then you should know about $\log_b t$. You should also know that these two equations are equivalent:

$$b^r = t, \qquad \log_b t = r.$$

(The number b is called the *base*.)

Properties of exponents and logarithms

Fix a positive number b; on the left-hand side below are properties of exponents that hold for any real numbers r and s and any positive number t. On the right-hand side below, suppose that $b \neq 1$ (and still positive); these properties of logarithms hold for any real number r and any positive numbers s and t:

$$\begin{aligned}
b^r &> 0; & \log_b (b^r) &= r; \\
b^0 &= 1; & \log_b 1 &= 0; \\
b^{r+s} &= b^r b^s; & \log_b (st) &= \log_b s + \log_b t; \\
b^{-s} &= \frac{1}{b^s}; & \log_b \left(\frac{1}{t}\right) &= -\log_b t; \\
b^{r-s} &= \frac{b^r}{b^s}; & \log_b \left(\frac{s}{t}\right) &= \log_b s - \log_b t; \\
b^1 &= b; & \log_b b &= 1; \\
b^{rs} &= (b^s)^r; & \log_b (s^r) &= r \log_b s; \\
b^{1/t} &= \sqrt[t]{b}; & \log_b \left(\sqrt[t]{b}\right) &= \frac{1}{t}; \\
b^{r/t} &= \sqrt[t]{b^r}; & \log_b \left(\sqrt[t]{s}\right) &= \frac{\log_b s}{t}.
\end{aligned}$$

In addition, there are these change-of-base rules:

- If b and e are positive, $e \neq 1$, and r is any real number, then $b^r = e^{r \log_e b}$;
- if b, e, and t are positive, $b \neq 1$, and $e \neq 1$, then $\log_b t = \dfrac{\log_e t}{\log_e b}$.

So in a way, you only need to know about exponents and logarithms for one specific base e. In calculus, it's convenient to choose the special constant $\mathrm{e} \approx 2.7$; we write $\ln t$ for $\log_{\mathrm{e}} t$.

So you should know the algebraic rules for exponents and logarithms with base e, and how to convert any other expression into base e:

$$b^r = \mathrm{e}^{r \ln b}; \qquad \log_b t = \frac{\ln t}{\ln b}.$$

(Here, b and t are positive, and $b \neq 1$.)

Rules for differentiation

There are two basic ways to go about differentiation expressions with exponentials and logarithms.

One way is to turn everything into a more complicated expression where the only base is e and learn rules for those expressions only:

$$\mathrm{d}(\mathrm{e}^u) = \mathrm{e}^u \,\mathrm{d}u; \qquad \mathrm{d}(\ln u) = \frac{\mathrm{d}u}{u}.$$

These are the simplest rules to learn, but before you can use them, you may have to do some algebra to change the base.

The other way is to learn rules for any exponential or logarithmic expression:

$$\mathrm{d}(v^u) = v^{u-1}(v \ln v \,\mathrm{d}u + u \,\mathrm{d}v); \qquad \mathrm{d}(\log_v u) = \frac{v \ln v \,\mathrm{d}u - u \ln u \,\mathrm{d}v}{uv(\ln v)^2}.$$

These are more complicated, but you can always use them right away.

It may also be handy to learn the special cases of the above rules when one part of the expression is constant (which can then be simplified a bit):

$$\mathrm{d}(v^c) = cv^{c-1}\,\mathrm{d}v; \qquad \mathrm{d}(\log_v c) = -\frac{\ln c \,\mathrm{d}v}{v(\ln v)^2};$$

$$\mathrm{d}(c^u) = c^u \ln c \,\mathrm{d}u; \qquad \mathrm{d}(\log_c u) = \frac{\mathrm{d}u}{u \ln c}.$$

The first of these four is our old friend the Power Rule; it's the *only* one out of all of the rules above that applies when the base v is not always positive.

Rules for limits

You can take limits of exponential and logarithmic expressions by simply plugging in the limiting value, as usual, so long as the base of every such expression is *positive*. But even when the original expressions are positive, it's possible that the limit of the base is zero, so watch out!

In other cases, I advise you to change the base to e and use these rules to deal with infinities:

$$\mathrm{e}^{\infty} = \infty; \qquad \ln \infty = \infty;$$

$$\mathrm{e}^{-\infty} = 0; \qquad \ln 0 = -\infty.$$

It's possible to learn more complicated rules for arbitrary bases, but there are far too many than I would want to handle.

To handle indeterminate forms, we can use L'Hôpital's Rule. In the case of a logarithm, L'Hôpital will apply directly if it's indeterminate; in the case of an exponential expression, use

$$b^r = \mathrm{e}^{r \ln b} = \mathrm{e}^{\frac{\ln b}{1/r}}$$

and apply L'Hôpital to the exponent. Note especially the indeterminate form 0^0. (Every other indeterminate form involves infinity somewhere.) Since the base 0 is not positive, you *cannot* simply evaluate this to 1.