Here is a list of most of the rules of differentiation that you will need in Applied Calculus. (There are a few more rules, involving logarithms, that we will cover later; they're listed on the back.) The rules marked with asterisks are the ones that you absolutely must know, but the others are also handy.

In the rules below, u and v stand for variable quantities, which may be complicated expressions involving many variables (or may be very simple). On the other hand, k stands for a constant, which will usually be a simple real number (but could be something more complicated that evaluates to a constant). Also, a technicality about undefined operations (such as division by zero): what these rules state is that *if* the right-hand side is defined, then so is the left-hand side and the two sides are equal.

Each rule is given with an example. In the example, the first step is the application of the rule, but there may be more steps given by applying other rules or basic algebra.

	Rule	Example
*	$\mathrm{d}k = 0$	$\mathrm{d}(6) = 0$
*	$\mathrm{d}\big(u^k\big) = ku^{k-1}\,\mathrm{d}u$	$\mathrm{d}(x^5) = 5x^{5-1}\,\mathrm{d}x = 5x^4\,\mathrm{d}x$
	$\mathrm{d}(\sqrt[k]{u}) = \dfrac{\sqrt[k]{u}\,\mathrm{d}u}{ku}$	$\mathrm{d}(\sqrt{x}) = \dfrac{\sqrt{x}\,\mathrm{d}x}{2x}$
*	$\mathrm{d}(uv) = v\,\mathrm{d}u + u\,\mathrm{d}v$	$\mathrm{d}(x^2y^3) = y^3\,\mathrm{d}(x^2) + x^2\,\mathrm{d}(y^3) = y^3(2x\,\mathrm{d}x) + x^2(3y^2\,\mathrm{d}y) = 2xy^3\,\mathrm{d}x + 3x^2y^2\,\mathrm{d}y$
	$\mathrm{d}(ku) = k\,\mathrm{d}u$	$\mathrm{d}(3p^4) = 3\,\mathrm{d}(p^4) = 3(4p^3\,\mathrm{d}p) = 12p^3\,\mathrm{d}p$
	$\mathrm{d}\left(\dfrac{u}{v}\right) = \dfrac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$	$\mathrm{d}\left(\dfrac{x+5}{x-4}\right) = \dfrac{(x-4)\,\mathrm{d}(x+5) - (x+5)\,\mathrm{d}(x-4)}{(x-4)^2} = \dfrac{(x-4)\,\mathrm{d}x - (x+5)\,\mathrm{d}x}{(x-4)^2} = -\dfrac{9\,\mathrm{d}x}{(x-4)^2}$
	$\mathrm{d}\left(\dfrac{u}{k}\right) = \dfrac{\mathrm{d}u}{k}$	$\mathrm{d}\left(\dfrac{x^2}{4}\right) = \dfrac{\mathrm{d}(x^2)}{4} = \dfrac{2x\,\mathrm{d}x}{4} = \dfrac{x\,\mathrm{d}x}{2}$
	$\mathrm{d}\left(\dfrac{k}{u}\right) = -\dfrac{k\,\mathrm{d}u}{u^2}$	$\mathrm{d}\left(\dfrac{1}{x}\right) = -\dfrac{1\,\mathrm{d}x}{x^2} = -\dfrac{\mathrm{d}x}{x^2}$
*	$\mathrm{d}(u+v) = \mathrm{d}u + \mathrm{d}v$	$\mathrm{d}(x^2+3x) = \mathrm{d}(x^2) + \mathrm{d}(3x) = 2x\,\mathrm{d}x + 3\,\mathrm{d}x = (2x+3)\,\mathrm{d}x$
	$\mathrm{d}(u+k) = \mathrm{d}u$	$\mathrm{d}(x+3) = \mathrm{d}x$
	$\mathrm{d}(-u) = -\mathrm{d}u$	$\mathrm{d}(-x^2) = -\mathrm{d}(x^2) = -2x\,\mathrm{d}x$
	$\mathrm{d}(u-v) = \mathrm{d}u - \mathrm{d}v$	$\mathrm{d}(2x-3y) = \mathrm{d}(2x) - \mathrm{d}(3y) = 2\,\mathrm{d}x - 3\,\mathrm{d}y$
	$\mathrm{d}(u-k) = \mathrm{d}u$	$\mathrm{d}(x^2-4) = \mathrm{d}(x^2) = 2x\,\mathrm{d}x$
	$\mathrm{d}(k-u) = -\mathrm{d}u$	$\mathrm{d}(3-2y) = -\mathrm{d}(2y) = -2\,\mathrm{d}y$
*	$\mathrm{d}\big(f(u)\big) = f'(u)\,\mathrm{d}u$	$\mathrm{d}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)\,\mathrm{d}\big(g(x)\big) = f'\big(g(x)\big)g'(x)\,\mathrm{d}x$

Notice: If you differentiate a scalar expression (one without the differential operator d in it) involving (say) the variables x, y, and z, then the result can always be put in the form

$$(\text{something})\,\mathrm{d}x + (\text{something})\,\mathrm{d}y + (\text{something})\,\mathrm{d}z,$$

where each (something) is again a scalar expression (no d). If it doesn't come out like that, then you did something wrong!

Here are the rules involving logarithms. Using the identities $u^v = \exp(v \ln u)$ and $\log_u v = \ln v / \ln u$, you only need the two rules marked with asterisks. (Here, $\exp u$ means e^u, where $\mathrm{e} \approx 2.718$ is the base of natural logarithms.)

$$* \quad \mathrm{d}(\exp u) = \exp u \, \mathrm{d}u \qquad \mathrm{d}\big(\exp(x-2)\big) = \exp(x-2)\,\mathrm{d}(x-2) = \exp(x-2)\,\mathrm{d}x$$

$$\mathrm{d}(k^u) = k^u \ln k \, \mathrm{d}u \qquad \mathrm{d}(2^x) = 2^x \ln 2 \, \mathrm{d}x$$

$$\mathrm{d}(u^v) = u^{v-1}(v \, \mathrm{d}u + u \ln u \, \mathrm{d}v) \qquad \mathrm{d}(x^x) = x^{x-1}(x \, \mathrm{d}x + x \ln x \, \mathrm{d}x) = x^x(1 + \ln x)\,\mathrm{d}x$$

$$* \quad \mathrm{d}(\ln u) = \frac{\mathrm{d}u}{u} \qquad \mathrm{d}\big(\ln(x^2+1)\big) = \frac{\mathrm{d}(x^2+1)}{x^2+1} = \frac{2x\,\mathrm{d}x}{x^2+1} = \frac{2x}{x^2+1}\,\mathrm{d}x$$

$$\mathrm{d}(\log_k u) = \frac{\mathrm{d}u}{u \ln k} \qquad \mathrm{d}(\log_3 x) = \frac{\mathrm{d}x}{x \ln 3} = \frac{1}{\ln 3\, x}\,\mathrm{d}x$$

$$\mathrm{d}(\log_u k) = -\frac{\ln k \, \mathrm{d}u}{u \ln^2 u} \qquad \mathrm{d}(\log_{p-2} \mathrm{e}) = -\frac{\ln \mathrm{e}\,\mathrm{d}(p-2)}{(p-2)\ln^2(p-2)} = -\frac{\mathrm{d}p}{(p-2)\ln^2(p-2)} = -\frac{1}{(p-2)\ln^2(p-2)}\,\mathrm{d}p$$

$$\mathrm{d}(\log_u v) = \frac{u \ln u \, \mathrm{d}v - v \ln v \, \mathrm{d}u}{uv \ln^2 u} \qquad \mathrm{d}\big(\log_x(x+1)\big) = \frac{x \ln x \, \mathrm{d}(x+1) - (x+1)\ln(x+1)\,\mathrm{d}x}{x(x+1)\ln^2 x} = \frac{x \ln x - x \ln(x+1) - \ln(x+1)}{x(x+1)\ln^2 x}\,\mathrm{d}x$$

To fill out the page, here are some more rules (without examples now) that we won't need at all in this course, just in case you ever find that you need them in your life. (It's all right if you don't have any idea what they're talking about; the last one on the left should make sense to you by the end of the course, but we're still not going to use it.)

$$* \quad \mathrm{d}(\sin u) = \cos u \, \mathrm{d}u \qquad \mathrm{d}(\operatorname{asin} u) = \frac{\mathrm{d}u}{\sqrt{1-u^2}}$$

$$\mathrm{d}(\cos u) = -\sin u \, \mathrm{d}u \qquad \mathrm{d}(\operatorname{acos} u) = -\frac{\mathrm{d}u}{\sqrt{1-u^2}}$$

$$\mathrm{d}(\tan u) = \frac{\mathrm{d}u}{\cos^2 u} \qquad \mathrm{d}(\operatorname{atan} u) = \frac{\mathrm{d}u}{u^2+1}$$

$$\mathrm{d}(\sinh u) = \cosh u \, \mathrm{d}u \qquad \mathrm{d}(\operatorname{asinh} u) = \frac{\mathrm{d}u}{\sqrt{u^2+1}}$$

$$\mathrm{d}(\cosh u) = \sinh u \, \mathrm{d}u \qquad \mathrm{d}(\operatorname{acosh} u) = \frac{\mathrm{d}u}{\sqrt{u^2-1}}$$

$$\mathrm{d}(\tanh u) = \frac{\mathrm{d}u}{\cosh^2 u} \qquad \mathrm{d}(\operatorname{atanh} u) = \frac{\mathrm{d}u}{1-u^2}$$

$$* \quad \mathrm{d}\Big(\int_u^v f(t)\,\mathrm{d}t\Big) = f(v)\,\mathrm{d}v - f(u)\,\mathrm{d}u \qquad \mathrm{d}\big(f(u,v)\big) = \frac{\partial f(u,v)}{\partial u}\,\mathrm{d}u + \frac{\partial f(u,v)}{\partial v}\,\mathrm{d}v$$