

Here is a list of the algebraic identities that are behind the various rules for adding fractions, multiplying negative numbers, and the like, as well as calculating in scientific notation. It's not a complete list (no list could ever be complete), but it's more complete than the lists in the textbook, and it should be complete enough.

The list covers addition, taking opposites, subtraction, multiplication, taking reciprocals, and division, all for real numbers. In addition, the list covers raising a real number to the power of an integer. But note that the reciprocal of zero, division of any real number by zero, and raising zero to the power of negative number are all undefined. I use  $a, b, c, d$  for real numbers and  $m, n$  for integer exponents.

Each of these is given with an example, which hopefully will help it make more sense.

$$\begin{array}{ll}
 -(-a) = a & -(-4) = 4 \\
 a + (-b) = a - b & 5 + (-2) = 5 - 2 = 3 \\
 a + (-b) = -(b - a) & 4 + (-6) = -(6 - 4) = -2 \\
 (-a) + b = b - a & (-3) + 7 = 7 - 3 = 4 \\
 (-a) + b = -(a - b) & (-4) + 3 = -(4 - 3) = -1 \\
 (-a) + (-b) = -(a + b) & (-3) + (-5) = -(3 + 5) = -8 \\
 a(-b) = -(ab) & 3 \cdot (-4) = -(3 \cdot 4) = -12 \\
 (-a)b = -(ab) & (-5) \cdot 2 = -(5 \cdot 2) = -10 \\
 (-a)(-b) = ab & (-3) \cdot (-2) = 3 \cdot 2 = 6 \\
 \frac{1}{-a} = -\frac{1}{a} \text{ if } a \neq 0 & \frac{1}{-2} = -\frac{1}{2} \\
 \frac{a}{1} = a & \frac{2}{1} = 2 \\
 \frac{a}{a} = 1 \text{ if } a \neq 0 & \frac{5}{5} = 1 \\
 \frac{ab}{b} = a \text{ if } b \neq 0 & \frac{6}{2} = \frac{3 \cdot 2}{2} = 3 \\
 \frac{b}{ab} = \frac{1}{a} \text{ if } a, b \neq 0 & \frac{3}{6} = \frac{3}{2 \cdot 3} = \frac{1}{2} \\
 \frac{ac}{bc} = \frac{a}{b} \text{ if } b, c \neq 0 & \frac{8}{12} = \frac{4 \cdot 2}{4 \cdot 3} = \frac{2}{3} \\
 \frac{0}{a} = 0 \text{ if } a \neq 0 & \frac{0}{7} = 0 \\
 \frac{a}{0} \text{ is undefined} & \frac{6}{0} \text{ is undefined}
 \end{array}$$

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ if $c \neq 0$	$\frac{2}{5} + \frac{4}{5} = \frac{2+4}{5} = \frac{6}{5}$
$a + \frac{b}{c} = \frac{ac+b}{c}$ if $c \neq 0$	$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \cdot 4 + 3}{4} = \frac{11}{4}$
$\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$ if $c, d \neq 0$	$\frac{2}{3} + \frac{4}{5} = \frac{2 \cdot 5 + 4 \cdot 3}{3 \cdot 5} = \frac{22}{15}$
$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$ if $c \neq 0$	$\frac{9}{7} - \frac{3}{7} = \frac{9-3}{7} = \frac{6}{7}$
$\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$ if $c, d \neq 0$	$\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$
$\frac{1}{a/b} = \frac{b}{a}$ if $a, b \neq 0$	$\frac{1}{5/4} = \frac{4}{5}$
$\frac{a/b}{c/d} = \frac{ad}{bc}$ if $b, c, d \neq 0$	$\frac{2/5}{3/4} = \frac{2 \cdot 4}{5 \cdot 3} = \frac{8}{15}$
$ab^n = \frac{a}{b}b^{n+1}$ if $b \neq 0$	$12 \times 10^5 = \frac{12}{10} \times 10^{5+1} = 1.2 \times 10^6$
$ab^n = (ab)b^{n-1}$ if $b \neq 0$	$0.45 \times 10^3 = (0.45 \cdot 10) \times 10^{3-1} = 4.5 \times 10^2$
$a^m a^n = a^{m+n}$ if $a \neq 0$	$10^3 \cdot 10^4 = 10^{3+4} = 10^7$
$\frac{a^m}{a^n} = a^{m-n}$ if $a \neq 0$	$\frac{10^9}{10^5} = 10^{9-5} = 10^4$
$ac + bc = (a + b)c$	$3 \text{ lb} + 4 \text{ lb} = (3 + 4) \text{ lb} = 7 \text{ lb}$
$a(bc) = (ab)c$	$5 \text{ ft} = 5(12 \text{ in}) = (5 \cdot 12) \text{ in} = 60 \text{ in}$
$(ac)(bc) = (ab)c^2$	$(3 \text{ m})(4 \text{ m}) = (3 \cdot 4) \text{ m}^2 = 12 \text{ m}^2$