Identities for simplifying algebraic expressions

Here is a list of algebraic identities in Beginning Algebra. It's not a complete list (no list could ever be complete), but it's more complete than the lists in the textbook, and it should be complete enough.

The list covers addition, taking opposites, subtraction, multiplication, taking reciprocals, and division, all for real numbers. In addition, the list covers raising a real number to the power of an integer. But note that the reciprocal of zero, division of any real number by zero, and raising zero to the power of negative number are all undefined. I use A, B, C, D for real numbers and m, n for integer exponents. Note that you can substitute *any* algebraic expression (a constant, a variable, or a more complicated expression) for one of these variables! (At least, you can as long as the result is defined and all exponents are integers.) That is usually what you're going to want to do.

Instead of giving examples with specific real numbers, I've used examples more relevant for algebra, using variables or other expressions, although I only use specific integers for exponents.

$$
\begin{array}{rl}
-(-A) = A & -(-2x) = 2x \\
-(A+B) = -A-B & -(3x+2) = -3x-2 \\
-(A-B) = -A+B & -(2y-4) = -2y+4 \\
A+(-B) = A-B & 3x+(-2y) = 3x-2y \\
A+B = B+A & 4+2x = 2x+4 \\
A-(B+C) = A-B-C & 2x-(3y+4) = 2x-3y-4 \\
A-(B-C) = A-B+C & 2x-(3y-4) = 2x-3y+4 \\
AC+BC = (A+B)C & 2x+3x = (2+3)x = 5x \\
AB+B = (A+1)B & 6p+p = (6+1)p = 7p \\
B+AB = (A+1)B & p+6p = (6+1)p = 7p \\
A+A = 2A & xy+xy = 2xy \\
AC-BC = (A-B)C & 3x-5x = (3-5)x = -2x \\
AB-B = (A-1)B & 3x-x = (3-1)x = 2x \\
A-A = 0 & 5x-5x = 0 \\
A+0 = A & 2x+3y-3y = 2x+0 = 2x \\
\dfrac{A}{B} = \dfrac{1}{B}A \text{ if } B \neq 0 & \dfrac{x}{2} = \dfrac{1}{2}x \\
\dfrac{AB}{C} = \dfrac{A}{C}B \text{ if } c \neq 0 & \dfrac{2t}{3} = \dfrac{2}{3}t \\
A(B+C) = AB+AC & 2(x+4) = 2x+2\cdot 4 = 2x+8 \\
A(B-C) = AB-AC & 3(5x-y) = 3\cdot 5x-3y = 15x-3y \\
(A+B)(C+D) = AC+AD+BC+BD & (x+2)(y^2+y) = xy^2+xy+2y^2+2y
\end{array}
$$

$$(AB)^n = A^n B^n \text{ if } A, B \neq 0 \text{ or } n \geq 0 \qquad (3x)^2 = 3^2 x^2 = 9x^2$$

$$\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n} \text{ if } B \neq 0 \text{ and } A \neq 0 \text{ or } n \geq 0 \qquad \left(\frac{x}{y}\right)^2 = \frac{x^3}{y^3} \text{ if } y \neq 0$$

$$A^0 = 1 \qquad (5t^2)^0 = 1$$

$$A^1 = A \qquad (6x)^1 = 6x$$

$$A^m A^n = A^{m+n} \text{ if } A \neq 0 \text{ or } m, n \geq 0 \qquad x^2 x^3 = x^{2+3} = x^5$$

$$AA^n = A^{n+1} \qquad xx^3 = x^{3+1} = x^4$$

$$A^n A = A^{n+1} \qquad x^3 x = x^{3+1} = x^4$$

$$\frac{A^m}{A^n} = A^{m-n} \text{ if } A \neq 0 \qquad \frac{y^6}{y^2} = y^{6-2} = y^4 \text{ if } y \neq 0$$

$$(A^m)^n = A^{mn} \text{ if } A \neq 0 \text{ or } m, n \geq 0 \qquad (x^2)^3 = x^{2 \cdot 3} = x^6$$

$$A^{-n} = \frac{1}{A^n}) \text{ if } A \neq 0 \qquad 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

$$\left(\frac{A}{B}\right)^{-n} = \frac{B^n}{A^n} \text{ if } A, B \neq 0 \qquad \left(\frac{2}{t}\right)^{-3} = \frac{t^3}{2^3} = t^3/8 \text{ if } t \neq 0$$

$$(-A)^n = A^n \text{ if } n \text{ is even and } A \neq 0 \text{ or } n \geq 0 \qquad (-x)^{-4} = x^{-4} = \frac{1}{x^4}$$

$$(-A)^n = -A^n \text{ if } n \text{ is odd and } A \neq 0 \text{ or } n \geq 0 \qquad (-x)^5 = -x^5$$