This exam ran from 11:00 to 12:20; if I wrote it well, then you didn't need the whole time. You may use any notes that you've written yourself, as well as quizzes that I've graded and returned to you, but not your textbook or anything else not written by you, and you may not communicate with anybody but me. You can come up and talk to me if you have questions, especially about the instructions. Also, you may use a calculator if you wish, although you shouldn't need one.

Take your time, and check over your answers. Read the instructions carefully, and be sure to show everything that they ask. You can always show *more* work if you like; for instance, you may draw a graph if you find it helpful, even when a problem does not require you to draw a graph. If you're unsure of your answer, then explain what you're unsure about, and show your work so that you can get as much partial credit as possible.

Don't forget to put your name on the exam!!!

1 *Factor the following polynomials completely. (Show at least one intermediate step for each, or explain how you know that it can't be factored.)*

a $3x^3 + 6x^2 + 3x$

First, I factor out the common factor of $3x$:

$$3x^3 + 6x^2 + 3x = 3x(x^2 + 2x + 1).$$

Next, I find two numbers which multiply to 1 and add to 2: 1 and 1 again. Thus,

$$3x^3 + 6x^2 + 3x = 3x(x^2 + 2x + 1) = 3x(x + 1)^2.$$

b $x^4 - 16y^4$

First, I factor the difference of the squares of x^2 and $4y^2$:

$$x^4 - 16y^4 = (x^2 - 4y^2)(x^2 + 4y^2).$$

Next, I factor the difference of the squares of x and $2y$:

$$x^2 - 4y^2 = (x - 2y)(x + 2y).$$

Thefore,

$$x^4 - 16y^4 = (x - 2y)(x + 2y)(x^2 + 4y^2).$$

c $3p^2 + 5p + 4$

These terms have no common factor. Since $3 \cdot 4 = 12$, I look for two numbers that multiply to 12 and add to 5. Since $1 + 12 = 13$, $2 + 6 = 8$, and $3 + 4 = 7$, this cannot be done. Therefore,

$$3p^2 + 5p + 4$$

cannot be factored.

2 *Simplify the following rational expressions; you may leave your answer either in factored or expanded form. (Show* at least *one intermediate step for each.)*

a $\frac{x^2-1}{x^2+5x+6} \cdot \frac{x^2-9}{x^2+3x+2}$

To multiply fractions, I factor everything top and bottom, combine, and cancel factors on opposite sides:

$$\frac{x^2-1}{x^2+5x+6} \cdot \frac{x^2-9}{x^2+3x+2} = \frac{(x-1)(x+1)\cdot(x-3)(x+3)}{(x+2)(x+3)\cdot(x+1)(x+2)} = \frac{(x-1)(x-3)}{(x+2)^2}.$$

b $\frac{x-\frac{1}{x}}{1+\frac{1}{x}}$

Since x is a common denominator for all of the fractions inside the fraction, I should multiply the entire top and bottom by x:

$$\frac{x-\frac{1}{x}}{1+\frac{1}{x}} = \frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1.$$

3 *Solve the following equations. (Show at least* two *intermediate steps for each.)*

a $3x+4=x^2$

I get every term on one side of the equation, then factor:

$$\begin{aligned} 3x+4 &= x^2; \\ 0 &= x^2-3x-4 \\ &= (x-4)(x+1); \\ x-4=0 &\text{ or } x+1=0; \\ x=4 &\text{ or } x=-1. \end{aligned}$$

b $\frac{1}{x+1}+\frac{1}{x-2}=\frac{1}{x^2-x-2}$

First I factor every denominator:

$$\begin{aligned} \frac{1}{x+1}+\frac{1}{x-2} &= \frac{1}{x^2-x-2}; \\ \frac{1}{x+1}+\frac{1}{x-2} &= \frac{1}{(x-2)(x+1)}. \end{aligned}$$

Now I see that $(x-2)(x+1)$ is a common denominator for the terms in this equation, so I multiply both sides by it:

$$\begin{aligned} \frac{1}{x+1}+\frac{1}{x-2} &= \frac{1}{(x-2)(x+1)}; \\ 1(x-2)+1(x+1) &= 1; \\ 2x-1 &= 1; \\ 2x &= 2; \\ x &= 1. \end{aligned}$$

If $x=1$, then neither $x+1$ nor $x-2$ is zero, so this answer is correct.

c **Extra credit:** $\frac{2}{t-1} + \frac{3}{t+1} + \frac{6}{t^2-1} = 0$

First I factor every denominator:

$$\begin{aligned} \frac{2}{t-1} + \frac{3}{t+1} + \frac{6}{t^2-1} &= 0; \\ \frac{2}{t-1} + \frac{3}{t+1} + \frac{6}{(t-1)(t+1)} &= 0. \end{aligned}$$

Now I see that $(t-1)(t+1)$ is a common denominator for the terms in this equation, so I multiply both sides by it:

$$\begin{aligned} \frac{2}{t-1} + \frac{3}{t+1} + \frac{6}{(t-1)(t+1)} &= 0; \\ 2(t+1) + 3(t-1) + 6 &= 0; \\ 5t + 5 &= 0; \\ 5t &= -5; \\ t &= -1. \end{aligned}$$

However, if $t = -1$, then $t + 1 = 0$, so this solution is extraneous; the correct answer is that there is **no solution**.

4 *Solve* one *of the following problems. (Show* at least *an equation that you can use to solve the problem, and be sure to include appropriate units in your final answer.)* **Extra credit:** *Solve both.*

a *A rectangular hallway is four times as long as it is wide. Its area is* 64 *square feet. What are its length and width?*

Let w be the hallway's width in feet, so that the length is $4w$ feet. Then the area is

$$w(4w) = 64$$

square feet, so $w = 4$ or $w = -4$. Of course, a negative width makes no sense, so the width is

$$4\,\mathrm{ft},$$

and the length is

$$4(4\,\mathrm{ft}) = 16\,\mathrm{ft}.$$

b *Suppose that it takes you* 5 *minutes to do an algebra problem, but it takes me only* 2 *minutes to do a problem. If we split up a test with* 49 *problems and work on them at the same time, then how fast can we finish it?*

Let t be the time in minutes that it takes us to do the test. To finish the test in that time, we must have a combined speed of $\frac{49}{t}$ problems per minute. Now, you do problems at a rate of $1/5$ problem per minute, and I do them at a rate of $1/2$ problem per minute. Thus, our combined speed is

$$\frac{1}{5} + \frac{1}{2} = \frac{49}{t},$$

so $t = 70$. Therefore it will take us 70 **minutes** to finish the test.

5 *Consider the equation $y = 3x^2$, and consider its graph in the (x, y)-plane.*

a *Is the point $(2, 12)$ on this graph? (Show what calculation you make to decide.)*

I put 2 in for x and 12 in for y:

$$y = 3x^2;$$
$$12 = 3(2)^2;$$
$$12 = 12.$$

This is true, so $(2, 12)$ **is on the graph**.

b *Is the point $(3, 18)$ on this graph? (Show what calculation you make to decide.)*

I put 3 in for x and 18 in for y:

$$y = 3x^2;$$
$$18 = 3(3)^2;$$
$$18 = 27.$$

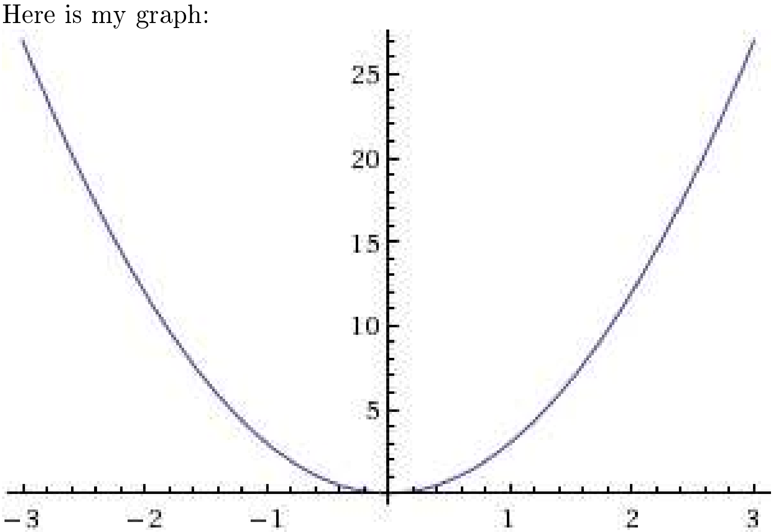
This is false, so $(2, 12)$ **is not on the graph**.

c *Draw a graph of this equation; try to make it clear, and be sure to label the scale. (To show your work, include a table of values that you calculate.)*

Here is my table of values:

x	$y = 3x^2$
-2	$3(-2)^2 = 12$
-1	$3(-1)^2 = 3$
0	$3(0)^2 = 0$
1	$3(1)^2 = 3$
2	$3(2)^2 = 12$

Here is my graph:



(This graph was produced by Wolfram Alpha. There really should be arrows at the ends of the curve.)