Simplify the following expressions, producing a rational expression (or a polynomial) in either expanded or factored form (your choice). Show at least two *intermediate steps for each.*

1 $\dfrac{\dfrac{1}{n+3}}{\dfrac{8n}{2n+6}}$

Since each side of the complex fraction is a single fraction, this is just a division problem:

$$\frac{\dfrac{1}{n+3}}{\dfrac{8n}{2n+6}} = \frac{1}{n+3} \div \frac{8n}{2n+6} = \frac{1}{n+3} \cdot \frac{2n+6}{8n} = \frac{1}{n+3} \cdot \frac{2(n+3)}{2^3 n} = \frac{2(n+3)}{2^3 n(n+3)} = \frac{1}{2^2 n} = \frac{1}{4n}.$$

2 $\dfrac{\dfrac{3}{y} - 1}{\dfrac{9}{y} - y}$

There are two ways to do this problem, and I'll show both.

One way is to simplify both the top and bottom of the complex fraction, turning it into a division problem like Problem 1:

$$\frac{3}{y} - 1 = \frac{3}{y} - \frac{y}{y} = \frac{3-y}{y} = \frac{-y+3}{y} = -\frac{y-3}{y},$$

and

$$\frac{9}{y} - y = \frac{9}{y} - \frac{y^2}{y} = \frac{9-y^2}{y} = \frac{-y^2+9}{y} = -\frac{y^2-9}{y} = -\frac{(y-3)(y+3)}{y},$$

so

$$\frac{\dfrac{3}{y} - 1}{\dfrac{9}{y} - y} = \frac{-\dfrac{y-3}{y}}{-\dfrac{(y-3)(y+3)}{y}} = \frac{\dfrac{y-3}{y}}{\dfrac{(y-3)(y+3)}{y}} = \frac{y-3}{y} \div \frac{(y-3)(y+3)}{y}$$
$$= \frac{y-3}{y} \cdot \frac{y}{(y-3)(y+3)} = \frac{y(y-3)}{y(y-3)(y+3)} = \frac{1}{y+3}.$$

Another way is to find a common multiple of the denominators of the simple fractions and multiply both sides of the complex fraction by it:

$$\frac{\dfrac{3}{y} - 1}{\dfrac{9}{y} - y} = \frac{y\left(\dfrac{3}{y} - 1\right)}{y\left(\dfrac{9}{y} - y\right)} = \frac{\dfrac{3y}{y} - y}{\dfrac{9y}{y} - y^2} = \frac{3-y}{9-y^2} = \frac{-y+3}{-y^2+9} = \frac{-(y-3)}{-(y^2-9)} = \frac{y-3}{y^2-9} = \frac{y-3}{(y-3)(y+3)} = \frac{1}{y+3}.$$