Solve the following equations; show at least one intermediate step for each. Also check your answers, at least enough to avoid extraneous solutions; show what numerical calculations you make to check.

1 $\dfrac{8}{x+4} = \dfrac{12}{x-3}$

Since this equation is simply an equation between two fractions, I'll solve it by cross multiplying:

$$\begin{aligned} \frac{8}{x+4} &= \frac{12}{x-3}; \\ 8(x-3) &= 12(x+4); \\ 8x-24 &= 12x+48; \\ -4x-24 &= 48; \\ -4x &= 72; \\ x &= -18. \end{aligned}$$

Neither $x+4 = -18+4 = -14$ nor $x-3 = -18-3 = -21$ is zero, so this solution should work.

2 $\dfrac{5}{2} + \dfrac{1}{x} = 4$

There are two ways to do this problem, and I'll show both.

One way is to simplify both sides of the equation, turning it into a cross multiplication problem like Problem 1:

$$\frac{5}{2} + \frac{1}{x} = \frac{5x}{2x} + \frac{2}{2x} = \frac{5x+2}{2x},$$

and

$$4 = \frac{4}{1};$$

so I get

$$\begin{aligned} \frac{5}{2} + \frac{1}{x} &= 4; \\ \frac{5x+2}{2x} &= \frac{4}{1}; \\ (5x+2)1 &= 4(2x); \\ 5x+2 &= 8x; \\ -3x+2 &= 0; \\ -3x &= -2; \\ x &= \frac{2}{3}. \end{aligned}$$

Another way is to find a common multiple of the denominators of the terms and multiply both sides of the equation by it:

$$\begin{aligned}
\frac{5}{2}+\frac{1}{x} &= 4; \\
2x\left(\frac{5}{2}+\frac{1}{x}\right) &= 2x(4); \\
\frac{2\cdot 5x}{2}+\frac{2x}{x} &= 8x; \\
5x+2 &= 8x; \\
-3x+2 &= 0; \\
-3x &= -2; \\
x &= \frac{2}{3}.
\end{aligned}$$

In either case, neither 2 nor $x = 2/3$ is zero, so this solution should work.

3 $\dfrac{6}{x^2} = \dfrac{2}{x}$

As in Problem 1, I'll solve this equation by cross multiplying:

$$\begin{aligned}
\frac{6}{x^2} &= \frac{2}{x}; \\
6x &= 2x^2; \\
-2x^2+6x &= 0; \\
-2x(x-3) &= 0; \\
-2x = 0 \text{ or } x-3 &= 0; \\
x = 0 \text{ or } x &= 3.
\end{aligned}$$

If $x = 0$, then $x^2 = (0)^2 = 0$ is zero (and also $x = 0$), so this solution is extraneous; if $x = 3$, then neither $x^2 = (3)^2 = 9$ nor $x = 3$ is zero, so this solution should work. Therefore,

$$x = 3.$$