Factor these polynomials completely. Show at least one intermediate step in each part.

1 $3x^2 - 2x - 8$

The terms are in standard order and have no common factor. The coefficients on the first and last terms multiply to $3 \cdot -8 = -24$, so I want two numbers that multiply to -24 and add to -2. Here are my attempts:

$$1 + -24 = -23,$$
$$2 + -12 = -10,$$
$$3 + -8 = -5,$$
$$4 + -6 = -2.$$

So the numbers that I want are 4 and -6; I split up $-2x$ as $4x - 6x$ and factor by grouping:

$$3x^2 - 2x - 8 = 3x^2 + 4x - 6x - 8 = x(3x + 4) - 2(3x + 4) = (x - 2)(3x + 4).$$

2 $81m^2 - 16n^2$

The terms are in standard order and have no common factor. Since $81m^2 = (9m)^2$ and $16n^2 = (4n)^2$, I can factor this as a difference of squares:

$$81m^2 - 16n^2 = (9m - 4n)(9m + 4n).$$