Quiz 5 **MATH-1100-ES32** **2011 October 14**

Factor these polynomials completely. Show at least one intermediate step in each part.

1 $-45x^2 + 3xy + 6y^2$

The terms are in standard order. Since all of the coefficients are multiples of 3 and the first coefficient is negative, I can factor out -3:

$$-3(15x^2 - xy - 2y^2).$$

Now since $15 \cdot -2 = -30$, I want two numbers that multiply to -30 and add to -1:

$$1 + -30 = -29;$$
$$2 + -15 = -13;$$
$$3 + -10 = -7;$$
$$5 + -6 = -1.$$

Now I split up the middle term and factor by grouping:

$$15x^2 - xy - 2y^2 = 15x^2 + 5xy - 6xy - 2y^2 = 5x(3x + y) - 2y(3x + y) = (5x - 2y)(3x + y).$$

Therefore, the final result is

$$3(5x - 2y)(3x + y).$$

2 $75x^2 + 90x + 27$

Again I can factor out -3:

$$3(25x^2 + 30x + 9).$$

Now there are two ways to continue:

If I notice that $25x^2 = (5x)^2$ and $9 = 3^2$, then I might be suspicious that $30x = 2(5x)(3)$, which happens to be true. Then I recognise $25x^2 + 30x + 9$ as a perfect square, specifically the square of $5x + 3$. This gives me the answer immediately:

$$3(5x + 3)^2.$$

If I don't notice this, then I can continue as usual; since $25 \cdot 9 = 225$, I want two numbers that multiply to 225 and add to 30:

$$1 + 225 = 226;$$
$$3 + 75 = 78;$$
$$5 + 45 = 50;$$
$$15 + 15 = 30.$$

(Since my two numbers are the same, this also tells me that I have a perfect square, so I could go back and do that method now. But here I'll continue as usual.) So,

$$25x^2 + 30x + 9 = 25x^2 + 15x + 15x + 9 = 5x(5x + 3) + 3(5x + 3) = (5x + 3)(5x + 3) = (5x + 3)^2.$$

Therefore, the final result is

$$3(5x + 3)^2.$$

3 $x^4 - 1$

The terms are in standard order and have no common factor. Since there are two terms, I look for a difference of squares of cubes. Since $x^4 = (x^2)^2$ and $1 = 1^2$, I have a difference of squares:

$$x^4 - 1 = (x^2 - 1)(x^2 + 1).$$

The first factor is again a difference of squares, so I can factor further:

$$(x - 1)(x + 1)(x^2 + 1).$$

A sum of squares (with no common factor) is prime, so this is the final result.