Quiz 12

Матн-1150-es32

1 A rectangle in the (x, y)-plane has one corner in Quadrant I on the graph of

$$y = 16 - x^2,$$

another at the origin, and one on each axis. Express the area of the rectangle as a function of the x-coordinate of the point in Quadrant I that is a vertex of the rectangle.

The base of the rectangle has length x, and the height has length y, so the area is xy. Since $y = 16 - x^2$, this area is $x(16 - x^2) = -x^3 + 16x$. If A is the area of the rectangle with one corner at (x, y), then

$$A = -x^3 + 16x.$$

2 A wire is bent into the shape of a circle. Express the area of the circle as a function of the length of the wire. (Hint: If r is the radius of a circle, then the distance around the circle (its perimeter or circumference) is $2\pi r$, while the area is πr^2 .)

Let x be the length of the wire. Then the circumference of the circle is also x, so $x = 2\pi r$, which means that $r = \frac{x}{2\pi}$. Therefore, the area is $\pi \left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$. If A is this area, then

$$A = \frac{x^2}{4\pi}.$$