A farmer with 4000 metres of fencing wants to enclose a rectangular plot that borders on a river. The farmer does not fence the side along the river.

1 Write an expression for the area (in square metres) of the plot as a function of the length $x$ (in metres) of a side perpendicular to the river.

Let $y$ be the length of the side parallel to the river (in metres). Then the total length of fencing is $2 x+y$ :

$$
\begin{aligned}
2 x+y & =4000 \\
y & =4000-2 x .
\end{aligned}
$$

The area is $x y$ :

$$
A=x y=x(4000-2 x) .
$$

(In other words, $A=f(x)$, where $f(x)=x(4000-2 x)$.)
2 What should the length of the side perpendicular to the river be in order to enclose the maximum possible area? (Or if you prefer, give the maximum area. In either case, show what equation you solve or what numerical calculation you make, and be sure to include correct units of measurement in your final answer.)

The area $A$ is a quadratic function of $x$ :

$$
A=x(4000-2 x)=-2 x^{2}+4000 x .
$$

This is quadratic with $a=-2, b=4000$, and $c=0$, so

$$
h=-\frac{b}{2 a}=-\frac{4000}{2(-2)}=1000
$$

and

$$
k=f(h)=f(1000)=(1000)(4000-2(1000))=2000000 .
$$

Since $a<0$, this vertex marks the maximum value of the function, which is the maximum area that we want. Therefore, the maximum area occurs when the length is 1000 metres (and the maximum area is 2000000 square metres).

