Solve each system of equations completely. (Show at least enough work that I can tell which method you're using.)

1 $\begin{cases} 5x - y = 21 \\ 2x + 3y = -12 \end{cases}$

I'll solve this one by elimination; I multiply the first equation by 3 and then add them both together:

$$\begin{aligned} 15x - 3y &= 63, \\ +\ 2x + 3y &= -12; \\ \hline 17x \qquad &= 51; \\ x \qquad &= 3. \end{aligned}$$

Then I use this result in the first equation to solve for y:

$$\begin{aligned} 5(3) - y &= 21; \\ y &= -7. \end{aligned}$$

Therefore,

$$(x, y) = (3, -7).$$

2 $\begin{cases} x + 2y = 4 \\ 2x + 4y = 8 \end{cases}$

I'll solve this one by substitution, first solving the first equation for x:

$$\begin{aligned} x + 2y &= 4; \\ x &= -2y + 4. \end{aligned}$$

Now I can try to solve for y:

$$\begin{aligned} 2(-2y + 4) + 4y &= 8; \\ 8 &= 8. \end{aligned}$$

This came out as simply a true statement, so there is no unique solution; instead,

$$x = -2y + 4$$

is the only result. You could also solve for y and give the answer as

$$y = -\frac{1}{2}x + 2.$$

3 $\begin{cases} x - y - z = 1 \\ 2x + 3y + z = 2 \\ 3x + 2y = 0 \end{cases}$

I'll solve this one by elimination again; I multiply the last equation by -1 and then add them all together:

$$\begin{aligned} x - y - z &= 1, \\ 2x + 3y + z &= 2, \\ +\ -3x - 2y &= 0; \\ \hline 0 &= 3. \end{aligned}$$

This statement is false, so there is **no solution**.