Quiz 13 **Math-1150-es35** **2011 November 22**

1 *Break down*

$$\log_2\left(\frac{x^3}{x-3}\right)$$

into an expression involving logarithms of the simplest possible arguments. (You may assume that x is positive.)

Division inside the logarithm becomes subtraction outside, and raising to a power inside becomes multiplication by a coefficient outside.

$$\log_2\left(\frac{x^3}{x-3}\right) = \log_2(x^3) - \log_2(x-3) = 3\log_2 x - \log_2(x-3).$$

Since $x-3$ (and course x) can't be factored, this is as far as I can break it down.

2 *Combine*

$$2\log_2(x+1) - \log_2(x+3) - \log_2(x-1)$$

into a single logarithm.

Multiplication by a coefficient outside the logarithm becomes raising to a power inside it, addition outside becomes multiplication inside, and subtraction outside becomes division inside.

$$\begin{aligned} 2\log_2(x+1) - \log_2(x+3) - \log_2(x-1) &= \log_2\left[(x+1)^2\right] - \left[\log_2(x+3) + \log_2(x-1)\right] \\ &= \log_2\left[(x+1)^2\right] - \log_2\left[(x+3)(x-1)\right] = \log_2\left[\frac{(x+1)^2}{(x+3)(x-1)}\right]. \end{aligned}$$

3 *Solve the equation*

$$\log_2(x+7) + \log_2(x+8) = 1.$$

(Show at least one intermediate step.) Either leave the answer in exact form or round off to three decimal places.

I combine both sides into a single logarithm, then drop the logarithms:

$$\begin{aligned} \log_2(x+7) + \log_2(x+8) &= 1; \\ \log_2\left[(x+7)(x+8)\right] &= \log_2 2; \\ (x+7)(x+8) &= 2; \\ x^2 + 15x + 54 &= 0; \\ x = -9 &\text{ or } x = -6. \end{aligned}$$

However, I must check for extraneous solutions; if $x=-9$, then $x+7$ or $x+8$ is negative, and I can't take a logarithm of a negative number. If $x=-6$, however, then $x+7$ and $x+8$ are both positive, so this solution should work. Therefore,

$$x = -6.$$