4.4.3

a The revenue is

$$R = xp = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$$

dollars.

d Since R is a quadratic function of x with coefficients $a = -1/6$, $b = 100$, and $c = 0$, R is maximised (because $a < 0$) when x is

$$h = -\frac{b}{2a} = -\frac{100}{2(-1/6)} = 300.$$

Then

$$R = -\frac{1}{6}(300)^2 + 100(300) = 15\,000.$$

Therefore, sell **three hundred** items to obtain the maximum revenue of **fifteen thousand dollars**.

e When $x = 300$,

$$p = -\frac{1}{6}(300) + 100 = 50,$$

so set the price to **fifty dollars** to maximise revenue. (Then $R = xp = (300)(50) = 15\,000$ again.)

4.4.9 Let x be the distance from the river in metres, and let y be the distance along the river in metres. Then $2x + y = 4000$, so

$$y = 4000 - 2x.$$

Next, the area is

$$A = xy = x(4000 - 2x) = -2x^2 + 4000x$$

square metres. Since A is a quadratic function of x with coefficients $a = -2$, $b = 4000$, and $c = 0$, A is maximised (because $a < 0$) when x is

$$h = -\frac{b}{2a} = -\frac{4000}{2(-2)} = 1000.$$

Then

$$A = -2(1000)^2 + 4000(1000) = 2000\,000.$$

(Alternatively, $y = 4000 - 2(1000) = 2000$, and then $A = xy = (1000)(2000) = 2000\,000$.) Therefore, the maximum area is **two million square metres**.