In each of the following problems, show at least one intermediate step.

1 *Break down*

$$\log\left(\frac{x(x+2)}{(x+3)^2}\right)$$

into an expression involving logarithms of the simplest possible (prime) arguments. (You may assume that x is positive.)

Division inside the logarithm becomes subtraction outside, multiplication inside becomes addition outside, and raising to a power inside becomes multiplication by a coefficient outside.

$$\log\left(\frac{x(x+2)}{(x+3)^2}\right) = \log\big(x(x+2)\big) - \log\big((x+3)^2\big) = \log x + \log(x+2) - 2\log(x+3).$$

Since $x+2$ and $x+3$ (and course x) can't be factored, this is as far as I can break it down.

2 *Combine*

$$3\log_5 u + 4\log_5 v$$

into a single logarithm.

Multiplication by a coefficient outside the logarithm becomes raising to a power inside it, and addition outside becomes multiplication inside.

$$3\log_5 u + 4\log_5 v = \log_5 u^3 + \log_5 v^4 = \log_5 u^3 v^4.$$

3 *Solve the equation*

$$\log_2(x+7) + \log_2(x+8) = 1.$$

Either leave the answer in exact form or round off to three decimal places.

I combine both sides into a single logarithm, then drop the logarithms:

$$\begin{aligned}
\log_2(x+7) + \log_2(x+8) &= 1;\\
\log_2[(x+7)(x+8)] &= \log_2 2;\\
(x+7)(x+8) &= 2;\\
x^2 + 15x + 54 &= 0;\\
x = -9 &\text{ or } x = -6.
\end{aligned}$$

However, I must check for extraneous solutions; if $x=-9$, then $x+7$ or $x+8$ is negative, and I can't take a logarithm of a negative number. If $x=-6$, however, then $x+7$ and $x+8$ are both positive, so this solution should work. Therefore,

$$x = -6.$$

4 *Solve the equation*

$$1.2^x = (0.5)^{-x}.$$

Either leave the answer in exact form or round off to three decimal places.

To solve this, I take logarithms base 1.2 and break down the result, then solve for x like normal:

$$\begin{aligned}
1.2^x &= (0.5)^{-x};\\
\log_{1.2} 1.2^x &= \log_{1.2} 0.5^{-x};\\
x &= -x\log_{1.2} 0.5;\\
x + x\log_{1.2} 0.5 &= 0;\\
(1+\log_{1.2} 0.5)x &= 0;\\
x &= 0.
\end{aligned}$$

The last step relies on knowing that $1+\log_{1.2} 0.5 \neq 0$, which can be verified with a caclulator (among other ways).