1 *Suppose that you run a business; according to your market research, if you sell widgets at a a price of p dollars each, then you'll be able to sell*

$$x = -20p + 500$$

widgets per week.

a *Write an expression for the total revenue (in dollars) as a function of either x or p.*

Revenue is price times quantity:

$$R = xp = (-20p + 500)p = -20p^2 + 500p.$$

(In other words, $R = f(x)$, where $f(x) = -20p^2 + 500p$.)

b *What price p will result in the maximum revenue? (Show what equation you solve or what numerical calculation you make, and be sure to include correct units of measurement in your final answer.)*

The revenue R is a quadratic function of the price p, with $a = -20$, $b = 500$, and $c = 0$, so

$$h = -\frac{b}{2a} = -\frac{500}{2(-20)} = \frac{25}{2};$$

since $a < 0$, this gives the maximum value of the function, which is the maximum revenue that we want. Since $25/2 = 12.5$, the maximum revenue occurs when **the price is** \$12.50.

2 *A farmer with 4000 metres of fencing wants to enclose a rectangular plot that borders on a river. The farmer does not fence the side along the river.*

a *Write an expression for the area (in square metres) of the plot as a function of the length (in metres) of one of the sides of the rectangle. (Indicate which one you use: the side parallel to the river or the side perpendicular to the river.)*

Let x be the length of the side perpendicular to the river (in metres), and let y be the length of the side parallel to the river (in metres). Then the total length of fencing is $2x + y$ metres:

$$2x + y = 4000;$$
$$y = 4000 - 2x.$$

The area is xy square metres:

$$A = xy = x(4000 - 2x) = -2x^2 + 4000x.$$

(In other words, $A = f(x)$, where $f(x) = -2x^2 + 4000x$.)

b *What is the largest possible area that the farmer can enclose? (Show what equation you solve or what numerical calculation you make, and be sure to include correct units of measurement in your final answer.)*

The area A is a quadratic function of x, with $a = -2$, $b = 4000$, and $c = 0$, so

$$h = -\frac{b}{2a} = -\frac{4000}{2(-2)} = 1000;$$

since $a < 0$, this gives the maximum value of the function, which is the maximum area that we want. Therefore, the maximum area occurs when the perendicular length is $x = 1000$ metres, so the parallel length is $y = 4000 - 2x = 2000$ metres, and **the maximum area is $xy = 2000\,000$ square metres**).