

1 Given

$$\begin{aligned}f(x) &= x^2, \\g(x) &= x^2 + 4,\end{aligned}$$

find a simplified formula for $g \circ f$. (Show at least one intermediate step.)

If it helps, write the formula for g as

$$g(y) = y^2 + 4.$$

Then taking y to be $f(x)$, we have

$$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)^2 + 4 = x^4 + 4.$$

2 Given

$$f(x) = 4x + 2,$$

find a formula for the inverse of f . Show at least what equation you solve to find this, as well as your final answer.

I set $f(x) = y$ and solve for x to find $f^{-1}(y)$:

$$\begin{aligned}f(x) &= y; \\4x + 2 &= y; \\4x &= y - 2; \\x &= \frac{1}{4}y - \frac{1}{2}; \\f^{-1}(y) &= \frac{1}{4}y - \frac{1}{2}.\end{aligned}$$

3 Suppose that g is a one-to-one function, the domain of g is $(-\infty, 0]$, and the range of g is $[0, \infty)$. State the domain and range of g^{-1} (indicating which is which).

The domain of g^{-1} is the same as the range of g :

$$\text{dom } g^{-1} = \text{ran } g = [0, \infty).$$

Similarly, the range of g^{-1} is the same as the domain of g :

$$\text{ran } g^{-1} = \text{dom } g = (-\infty, 0].$$