1 *A rectangle in the (x, y)-plane is inscribed (set tightly inside) a circle with the equation $x^2 + y^2 = 4$. Express the area of the rectangle as a function of the x-coordinate of the point in Quadrant I that is a vertex of the rectangle.*

Let x and y be the coordinates of this vertex. Since $x^2 + y^2 = 4$ and $y > 0$, I solve to get $y = \sqrt{4 - x^2}$. The base of the rectangle has length $l = 2x$, and the height has length $h = 2y$, so the area is $A = lh = (2x)(2y) = 4xy$. Therefore,

$$A = 4x\sqrt{4 - x^2}.$$

2 *From a square piece of cardboard with 24 inches on each side, you cut out a square piece from each corner (with each piece the same size), leaving behind a cross shape; think of this cross as a central square with a rectangle on each side, and fold up the rectangles to make a box with no lid. Express the volume of this box as a function of the length of the square piece cut out of one of the corners.*

Let x be the length in inches of a piece cut out, and let y be the remaining length in inches. Then $2x + y = 24$, so $y = 24 - 2x$. The length and width of the box are $l = y$ and $w = y$, while the height is $h = x$. Therefore, the volume is

$$V = lwh = y^2x = x(24 - 2x)^2.$$

3 *Consider the points $(3, -4)$ and $(5, 4)$ in the cartesian real number plane.*

a *What is the distance between these points? (Show what numerical calculation you make.)*

The distance is

$$\sqrt{\big((5) - (3)\big)^2 + \big((4) - (-4)\big)^2} = \sqrt{(2)^2 + (8)^2} = \sqrt{(4) + (64)} = \sqrt{68} = 2\sqrt{17}.$$

b *What is the midpoint between these points? (Show what numerical calculation you make.)*

The midpoint is

$$\left(\frac{(3) + (5)}{2}, \frac{(-4) + (4)}{2}\right) = \left(\frac{8}{2}, \frac{0}{2}\right) = (4, 0).$$