The final exam will be on September 23 Wednesday during the normal class period. It should take about an hour if you do it carefully, so we can spend the beginning of that class period on review. You may use one sheet of notes that you've written yourself, but not your textbook or anything else not written by you, and you may not communicate with anybody but me. Also, you may use a calculator if you wish, although you shouldn't really need one.

The answers to the questions in this exam are on the last page.

1 How is the graph of the equation

$$y^4 = 2x^2 - 3$$

symmetric?

a with respect to the x-axis

b with respect to the y-axis

c with respect to the origin

d all of the above

2 Is the function

$$f(x) = \frac{5x^4 - 3}{3x^2 - 2}$$

even or odd?

a even

b odd

c both

d neither

3 Find an equation in (x, y) for the line through the points $(-4, 0)$ and $(1, -5)$.

a $y = -x + 4$

b $y = x + 4$

c $y = x - 4$

d $y = -x - 4$

4 Find both intercepts of the line in the (x, y)-plane with the equation

$$3x - 2y = 5.$$

a $\left(\frac{5}{2}, 0\right)$ and $\left(0, -\frac{5}{3}\right)$

b $\left(-\frac{5}{2}, 0\right)$ and $\left(0, \frac{5}{3}\right)$

c $\left(-\frac{5}{3}, 0\right)$ and $\left(0, \frac{5}{2}\right)$

d $\left(\frac{5}{3}, 0\right)$ and $\left(0, -\frac{5}{2}\right)$

5 Consider these equations:

$$6x - 3y = -1,$$
$$x + 8y = 2.$$

How are their graphs related?

a parallel

b perpendicular

c both

d neither

6 Solve the system of equations

$$\begin{cases} -3x - 8y = -35, \\ -3x + 7y = 25. \end{cases}$$

a $(x, y) = (-1, -4)$

b $(x, y) = (1, -4)$

c $(x, y) = (1, 4)$

d $(x, y) = (-1, 4)$

7 Given

$$f(x) = \frac{x - 2}{x + 4},$$

find $f(0)$.

a $f(0) = 0$

b $f(0) = \frac{1}{2}$

c $f(0) = -\frac{1}{2}$

d $f(0) = 1$

8 Given

$$f(x) = \begin{cases} x + 3 & \text{for } x \leq 0, \\ x - 3 & \text{for } 0 < x < 1, \\ x - 2 & \text{for } x \geq 1, \end{cases}$$

find $f(1)$.

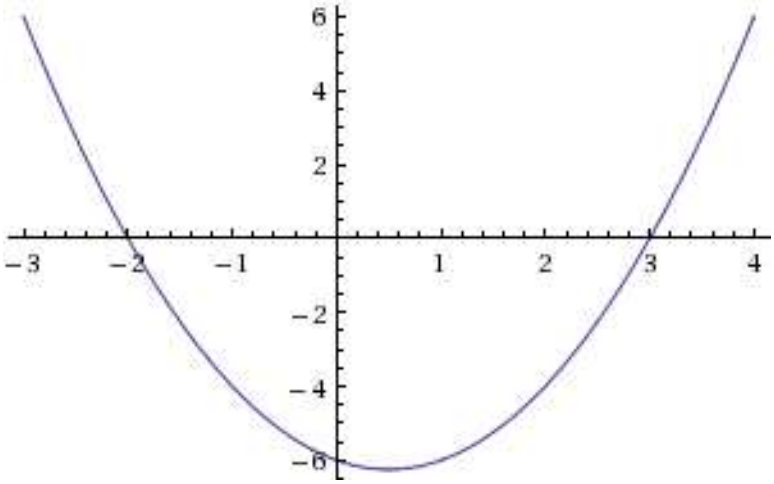
a $f(1) = 4$

b $f(1) = -2$

c $f(1) = -1$

d $f(1)$ is undefined

9 Given that f is the function with the graph below, find $f(2)$.



a $f(2) = -2.2$

b $f(2) = 3.2$

c $f(2) = -4$

d $f(2) = 0$

10 Given

$$f(x) = 3x + 6,$$
$$g(x) = 6x + 3,$$

write down a simplified formula for $f - g$.

a $(f-g)(x) = -3x^2 + 9x$

b $(f-g)(x) = -3x + 9$

c $(f-g)(x) = -3x + 3$

d $(f-g)(x) = -3x^2 + 3x$

11 Given

$$f(x) = x + 6,$$
$$g(x) = x^2,$$

write down a simplified formula for the composite function $f \circ g$.

a $(f \circ g)(x) = x^2 + 36$

b $(f \circ g)(x) = x^2 + 6$

c $(f \circ g)(x) = x^2 + 12x + 36$

d $(f \circ g)(x) = x^3 + 6x^2$

12 Given

$$f(x) = \frac{x+5}{5},$$

write down a simplified formula for the inverse function f^{-1}.

a $f^{-1}(x) = \dfrac{5}{x+5}$

b $f^{-1}(x) = \dfrac{5}{x-5}$

c $f^{-1}(x) = 5x + 5$

d $f^{-1}(x) = 5x - 5$

13 Given

$$g(x) = \sqrt{x-2},$$

what is the domain of g?

a $\{x \mid x > -2\} = (-2, \infty)$

b $\{x \mid x \geq 2\} = [2, \infty)$

c $\{x \mid x > 2\} = (2, \infty)$

d $\{x \mid x \geq -2\} = [-2, \infty)$

14 Given

$$g(x) = \sqrt{x-2},$$

what is the range of g?

a $\{x \mid x \geq -2\} = [-2, \infty)$

b $\{x \mid x \geq 0\} = [0, \infty)$

c $\mathbb{R} = (-\infty, \infty)$

d $\{x \mid x \geq 2\} = [2, \infty)$

15 Given

$$g(x) = \sqrt{x-2},$$

What is the average rate of change of g from 3 to 11?

a $\dfrac{1}{4}$

b 4

c $-\dfrac{1}{4}$

d -4

16 Given

$$f(x) = 3x^2 - 12x + 9,$$

what is the vertex of the graph of f?

a $(-2, 45)$

b $(0, 9)$

c $(1, 0)$

d $(2, -3)$

17 Solve the equation

$$2^{x+3} = 4^{2x}.$$

a $x = 0$

b $x = \ln 2 \approx 0.7$

c $x = \log_2 3 \approx 1.6$

d $x = 1$

18 Given

$$f(x) = 3x^2 - 12x + 9,$$

what are the roots (zeroes) of f, if any?

a 1

b 3

c both of the above

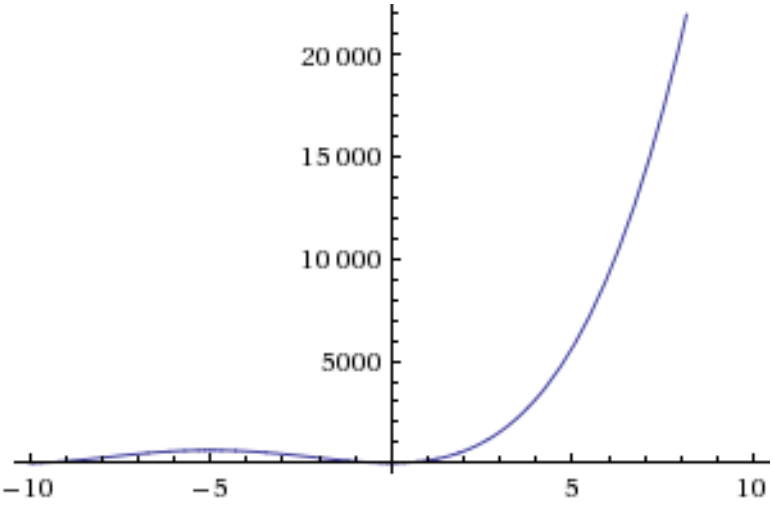
d neither of the above

19 Given

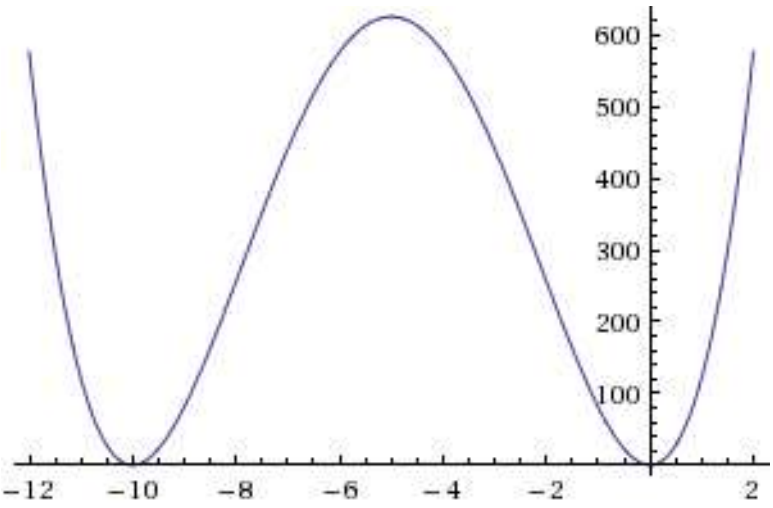
$$f(x) = x^4 + 20x^3 + 100x^2,$$

sketch a graph of f that shows all intercepts (if any), all turning points (if any), and the end behaviour.

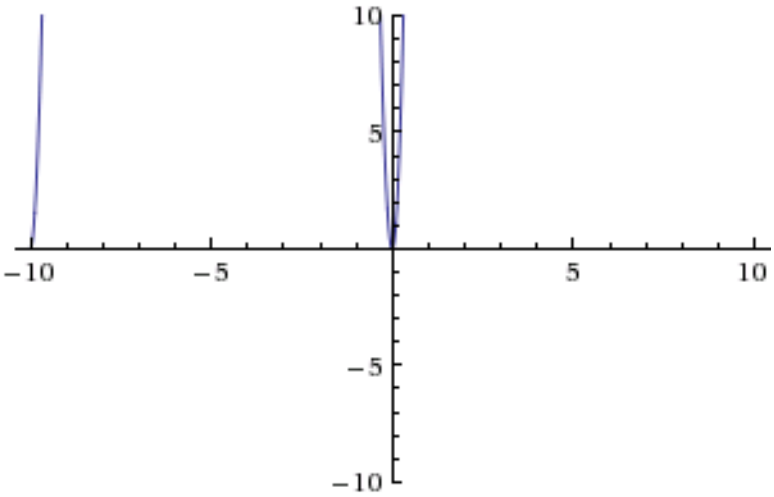
a



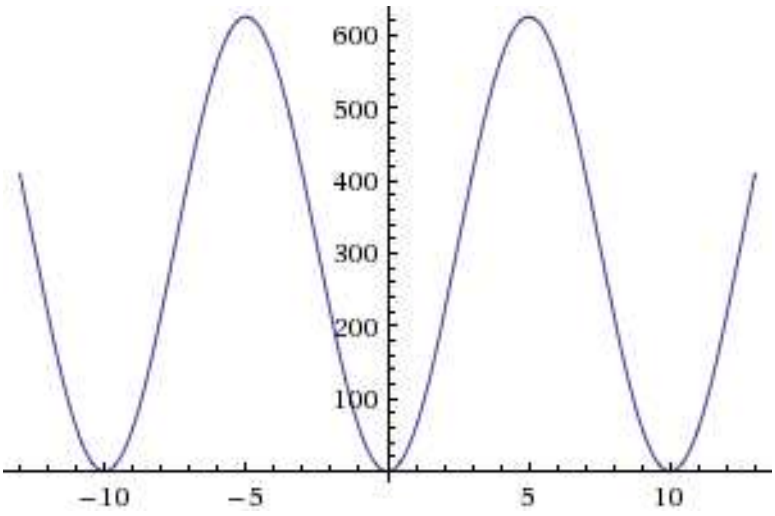
b


600
500
400
300
200
100
−12
−10
−8
−6
−4
−2
2

c


10
5
−10
−5
5
10
−5
−10

d


600
500
400
300
200
100
−10
−5
5
10

20 Given

$$f(x) = x^3 - 2x^2 - 4x + 8,$$

what is the multiplicity of the root (zero) 2?

a 0

b 1

c 2

d 3

21 Solve the equation

$$x^4 + x^3 + 2x^2 + 4x = 8$$

in the complex numbers.

a $x = 2$ or $x = -2\mathrm{i}$ or $x = -1$

b $x = -2$ or $x = 1$ or $x = -2\mathrm{i}$ or $x = 2\mathrm{i}$

c $x = 2\mathrm{i}$ or $x = -2$ or $x = 1$

d $x = 2$ or $x = -1$ or $x = 2\mathrm{i}$ or $x = -2\mathrm{i}$

22 Given

$$f(x) = \frac{x^2 - 4}{x^2 - 5x + 6},$$

what are the vertical asymptotes of f?

a $x = 3$

b $x = 2$

c both

d neither

23 Suppose that the unit price at which x items can be sold in a year is $1000 - x$ dollars. How many items should be sold in a year to maximise revenue? (Hint: First find a quadratic function for annual revenue as a function of the number of items, remembering that revenue is quantity times price.)

a 500

b 1000

c 250

d 300

24 While I was in graduate school, I didn't pay anything on my undergraduate loans. However, those loans continued to accrue interest at a 6% annual rate, compounded monthly. At the end of my six years of graduate school, I owed $20,000 (approximately) on these loans. What was the original amount of the loan (approximately) that I owed *before* I started graduate school?

a $13,000

b $14,000

c $18,000

d $19,000

25 Using a central-pivot irrigation system, a farmer irrigates a circular patch within a square field. The size of the field is fixed; the farmer irrigates the largest possible circle within that field. Express the area that the system irrigates as a function of the total area of the field.

Hints: The area of a circle is πr^2, where π is a constant (approximately 3.14) and r is the radius of the circle (the distance from its centre to its edge). The area of a square is l^2, where l is the length of any side of the square. A picture may help you see the relationship between the size of the circle (given by r) and the size of the square (given by l).

a $f(x) = \dfrac{\pi^2}{16}x^2 \approx 0.617x^2$

b $f(x) = \dfrac{\pi^2}{16}x \approx 0.617x$

c $f(x) = \dfrac{\pi}{4}x^2 \approx 0.785x^2$

d $f(x) = \dfrac{\pi}{4}x \approx 0.785x$

26 Fill in the blank: A point on a graph that is also on at least one of the coordinate axes is a(n) _____ of that graph.

27 If an equation in the variables x and y can be solved uniquely for y, then it defines y as a(n) _____ of x.

28 If $f(c) = 0$, then c is a(n) _____ of f.

29 If $f(a) = f(b)$ for all a and b, then f is a(n) _____ function.

30 A(n) _____ of a function is either a minimum or a maximum.

31 If $f(x) = mx + b$ for all x, then m is the _____ of f.

32 If $f\big(g(x)\big) = x$ for all x in the domain of g and $g\big(f(x)\big) = x$ for all x in the domain of f, then g is the _____ of f.

33 If $f(x) = ax^2 + bx + c$ for all x, then f is a(n) _____ function.

34 If $u = b^x$, $b > 0$, and $b \neq 1$, then x is the _____, base b, of u.

35 A line that a graph approaches arbitrarily closely but does not reach is a(n) _____ of that graph.

Answers

1 D, 2 A, 3 D, 4 D, 5 D, 6 C, 7 C, 8 C, 9 C, 10 C, 11 B, 12 D, 13 B, 14 B, 15 A, 16 D, 17 D, 18 C, 19 B, 20 C, 21 B, 22 A, 23 A, 24 B, 25 D.

26 intercept

27 function

28 root (or zero)

29 constant

30 extremum (or extreme value)

31 rate of change (*not* slope)

32 inverse (or inverse function)

33 quadratic

34 logarithm

35 asymptote