1 *Consider the function*

$$y = \cot\left(\frac{1}{4}x\right).$$

a *Give at least one point on its graph.*

The easiest might be to use $x = 0$; unfortunately, $\cot\left(\frac{1}{4}\cdot 0\right) = \cot 0$ is undefined. But since we can easily take (co)tangents of $\pi/4$, let's use $x = \pi$:

$$\cot\left(\frac{1}{4}\cdot\pi\right) = \cot\left(\frac{\pi}{4}\right) = 1.$$

Therefore, $(\pi, 1)$ is a point on the graph.

b *What is the period?*

The period of the (co)tangent function is π, so the period of this function is

$$\frac{\pi}{1/4} = 4\pi.$$

2 *Consider the sinusoidal function*

$$y = 4\sin(\pi x + 2) - 5.$$

a **Extra credit:** *What are its absolute maximum and minimum values?*

The average is $B = -5$, and the amplitude is $A = |4| = 4$, so the maximum is

$$M = B + A = -5 + 4 = -1,$$

and the minimum is

$$m = B - A = -5 - 4 = -9.$$

b *What is its period?*

Since the angular frequency is $\omega = \pi$, the period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2.$$

c *What is its phase shift?*

Since the angular frequency is not factored out, we are given directly $\phi = -2$. Therefore, the phase shift is

$$\psi = \frac{\phi}{\omega} = \frac{-2}{\pi} = -\frac{2}{\pi}.$$

3 *Write a formula for a sinusoidal function with amplitude 3, period 3π, and phase shift $-1/3$. (There are several possible answers to this question, but one whose average value is 0 is probably the easiest.)*

Since the period is $T = 3\pi$, the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3\pi} = \frac{2}{3}.$$

If I leave the angular frequency factored out, I can give the formula directly as

$$y = 3\sin\frac{2}{3}\left(x + \frac{1}{3}\right).$$

Alternatively, I can multiply the angular frequency and the phase shift to get

$$\phi = \omega\psi = \frac{2}{3}\left(-\frac{1}{3}\right) = -\frac{2}{9},$$

so

$$y = 3\sin\left(\frac{2}{3}x + \frac{2}{9}\right).$$