Evaluate (work out the value of) the following expressions; give exact results, not decimal approximations. (Show at least one intermediate step for each.)

1 $\arcsin \sin \frac{9\pi}{8}$, *or equivalently,* $\sin^{-1} \sin \frac{9\pi}{8}$

Since $9\pi/8 > \pi/2$, I'll try $9\pi/8 - 2\pi = -7\pi/8$, but $-7\pi/8 < -\pi/2$. So I'll try

$$\pi - \frac{9\pi}{8} = -\frac{\pi}{8}.$$

Since

$$-\frac{\pi}{2} \le -\frac{\pi}{8} \le \frac{\pi}{2},$$

that's the answer:

$$\arcsin \sin \frac{9\pi}{8} = -\frac{\pi}{8}.$$

2 $\tan \arcsin \frac{1}{3}$, *or equivalently,* $\tan \sin^{-1} \frac{1}{3}$

Let θ be $\arcsin(1/3)$; we want to find $\tan\theta$. Now, $\sin\theta = 1/3$ and $-\pi/2 \le \theta \le \pi/2$, so

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3},$$

so

$$\tan \arcsin \frac{1}{3} = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/3}{2\sqrt{2}/3} = \frac{\sqrt{2}}{4}.$$

3 $\operatorname{arccot} \sqrt{3}$, *or equivalently,* $\cot^{-1} \sqrt{3}$

Since

$$\cot \frac{\pi}{6} = \frac{\cos(\pi/6)}{\sin(\pi/6)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

and $0 \le \pi/6 \le \pi$,

$$\operatorname{arccot} \sqrt{3} = \frac{\pi}{6}.$$