1 *Solve*

$$\tan\theta = -\frac{\sqrt{3}}{3},$$

giving a general formula or formulas for all solutions.

First,

$$\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\arctan\left(\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6},$$

so the general solution is

$$\theta = -\frac{\pi}{6} + \pi k$$

for k any integer.

2 *Solve*

$$\sin 3\theta = -1$$

for $0 \leq \theta < 2\pi$.

I care about what 3θ can be:

$$0 \leq \theta < 2\pi;$$
$$0 \leq 3\theta < 6\pi.$$

In general,

$$3\theta = -\frac{\pi}{2} + 2\pi k$$

for k any integer; within this range,

$$3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}.$$

Therefore,

$$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}.$$

3 *Solve*

$$\sec\theta = -4$$

for $0 \leq \theta < 2\pi$. You may leave your answer in terms of inverse trigonometric operations, or you may use a calculator to find decimal approximations.

To use my calculator, I need the cosine instead of the secant:

$$\sec\theta = -4;$$
$$\frac{1}{\cos\theta} = -4;$$
$$\cos\theta = -\frac{1}{4}.$$

Within the stated range, then, my solutions are

$$\theta = \arccos\left(-\frac{1}{4}\right), 2\pi - \arccos\left(-\frac{1}{4}\right).$$

Using my calculator to approximate this,

$$\theta \approx 1.823, 4.460,$$

or

$$\theta \approx 104.5^\circ, 255.5^\circ.$$