Establish each identity. Show at least every step where you use a basic trigonemtric identity (wherever you change from one trigonometric operation to others).

1 $\cos\theta \cdot (\tan\theta + \cot\theta) = \csc\theta$

I first put the left-hand side in terms of sines and cosines:

$$\cos\theta \cdot (\tan\theta + \cot\theta) = \cos\theta \cdot \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right).$$

To make the remaining algebra easier, I'll use x for $\cos\theta$ and y for $\sin\theta$:

$$\cos\theta \cdot \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = x\left(\frac{y}{x} + \frac{x}{y}\right) = y + \frac{x^2}{y} = \frac{y^2 + x^2}{y}.$$

Wherever y^2 appears, I may change it to $1 - x^2$:

$$\frac{y^2 + x^2}{y} = \frac{1 - x^2 + x^2}{y} = \frac{1}{y}.$$

Finally, I get y out of the denominator, similar to rationalising it:

$$\frac{1}{y} = \frac{y}{y^2} = \frac{y}{1 - x^2}.$$

Now for the right-hand side, I do the same basic steps:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y} = \frac{y}{y^2} = \frac{y}{1 - x^2}.$$

Since both sides gave the same result, they are equal.

2 $\dfrac{\sec\theta - \csc\theta}{\sec\theta \cdot \csc\theta} = \sin\theta - \cos\theta$

I handle the left-hand side like I did before:

$$\frac{\sec\theta - \csc\theta}{\sec\theta \cdot \csc\theta} = \frac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}} = \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x}\frac{1}{y}} = \frac{y - x}{1} = y - x.$$

Then the right-hand side:

$$\sin\theta - \cos\theta = y - x.$$

These are the same, so they are equal.

3 **Extra credit:** $\ln\sec\theta = -\ln\cos\theta$

The left-hand side:

$$\ln\sec\theta = \ln\frac{1}{\cos\theta} = \ln\frac{1}{x} = \ln 1 - \ln x = 0 - \ln x = -\ln x.$$

The right-hand side:

$$-\ln\cos\theta = -\ln x.$$

These are the same, so they are equal.