

Evaluate (work out the value of) the following expressions; give exact results, not decimal approximations. (Show at least one intermediate step for each.)

1  $4 \cos 45^\circ - 2 \sin 45^\circ$

First,  $\sin 45^\circ = \sqrt{2}/2$ , and  $\cos 45^\circ = \sqrt{2}/2$  (the same). (If you get these from a diagram, then you might get them as  $1/\sqrt{2}$ , which you can then rationalise to  $\sqrt{2}/2$ .) Therefore,

$$4 \cos 45^\circ - 2 \sin 45^\circ = 4 \left( \frac{\sqrt{2}}{2} \right) - 2 \left( \frac{\sqrt{2}}{2} \right) = \sqrt{2}.$$

2  $\sec \frac{\pi}{4} + 2 \csc \frac{\pi}{3}$

First,  $\cos(\pi/4) = \sqrt{2}/2$ , so  $\sec(\pi/4) = 1/\cos(\pi/4) = 1/(\sqrt{2}/2) = \sqrt{2}$ . (You could also get this from a diagram as  $\sqrt{2}/1$ .) Next,  $\sin(\pi/3) = \sqrt{3}/2$ , so  $\csc(\pi/3) = 1/\sin(\pi/3) = 1/(\sqrt{3}/2) = 2/\sqrt{3}$ . (You could also get this from a diagram as  $2/\sqrt{3}$ .) Therefore,

$$\sec \frac{\pi}{4} + 2 \csc \frac{\pi}{3} = \sqrt{2} + 2 \left( \frac{2\sqrt{3}}{3} \right) = \sqrt{2} + \frac{4\sqrt{3}}{3}.$$

3  $\sin^2 30^\circ + \cos^2 60^\circ$

First,  $\sin 30^\circ = \sqrt{1}/2 = 1/2$ , and  $\cos 60^\circ = \sqrt{1}/2 = 1/2$  (the same). (You could also get these directly from a diagram.) Therefore,

$$\sin^2 30^\circ + \cos^2 60^\circ = (\sin 30^\circ)^2 + (\cos 60^\circ)^2 = \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 = \frac{1}{2}.$$