

The most general form for a sinusoidal function is

$$f(x) = A \sin \omega(x - \psi) + B,$$

where A , B , ω , and ψ are constant real numbers; in principle, A and ω should be positive. A graph of $y = f(x)$ is on the back page; go look at it. ... OK! that's a pretty complicated deal. So let's break it down bit by bit.

The 4 parameters

The number B is the **mean** of the function. It is the centre of the output values around which the function oscillates. Look at the graph on the back page and draw (dashed) the line $y = B$. Notice how it runs right through the middle of the graph. This is a characteristic feature of the mean.

Next, A is the **amplitude** of the function. It indicates how far the output values vary from the mean. The highest value that y will reach is $B + A$, and the lowest value that y will reach is $B - A$. Look at the graph and draw the lines $y = B + A$ and $y = B - A$. Notice how these are the highest and lowest that the graph reaches.

Next, ψ is the **phase shift** of the function. It indicates at what input argument the first cycle starts. If you calculate $f(\psi)$, you'll get B . So the output is the mean whenever the input is the phase shift. The cycle starts when the x -value is the phase shift ψ , and the y -value when the cycle starts is the mean B . If the function is given to you as $A \sin(\omega x - \phi) + B$, then the phase shift is $\psi = \phi/\omega$. Warning: There is a minus sign in front of ψ in the formula. So if you see a plus sign instead, then this indicates that ψ is negative.

Finally, ω is the **angular frequency** of the function. We use it to calculate the **period** T . The period is given by the formula $T = 2\pi/\omega$. This is how much the input changes, in either direction, before the function starts repeating. Since the first cycle begins at $x = \psi$, then it must end at $x = \psi + T$. Look at the the graph, and examine the points where the x -values are ψ , $\psi + T$, and $\psi - T$. Notice that the y -value is the mean B in each case.

Steps to drawing the graph

Draw some dashed lines at $y = B$, $y = B + A$, and $y = B - A$. This is sort of a layout for your graph. The line $y = B$ will run right down the middle of the graph, the line $y = B + A$ will run along the top of the graph, and the line $y = B - A$ will run along the bottom of the graph.

Now find the x -value where the first cycle starts; this is the phase shift ψ . Find the x -value where the first cycle ends; you get this from the phase shift and the period as $\psi + T$, or directly from the phase shift and the angular frequency as $\psi + 2\pi/\omega$. The y -values at these points will both be the mean B . So mark the points (ψ, B) and $(\psi + T, B)$.

The cycle begins by going up and to the right, then down, then back up to end. You know how high it must go, up to $B + A$, and how low it must go, down to $B - A$. You know where it starts, at (ψ, B) , and where it ends, at $(\psi + T, B)$. So draw it in.

Now you can copy this cycle to the left, where it will end at $(\psi - T, B)$. And you can copy it to the right, where it will end at $(\psi + 2T, B)$. Just repeat it off to the right and off to the left as far as you like. I drew the graph below using these steps.

Variations

I said above that A and ω should be positive. If you have a formula where they appear to be negative, you can change it to a formula where they're positive using these identities:

$$-\sin \theta = \sin(-\theta) = \sin(\theta + \pi).$$

Thus, if A is negative, then this effectively subtracts π from ϕ and therefore subtracts π/ω from the phase shift ψ ; if ω is negative, then the result is similar.

If you have a formula with a cosine instead of a sine, then use this identity:

$$\cos \theta = \sin\left(\theta + \frac{\pi}{2}\right).$$

Thus, using the cosine instead of the sine effectively subtracts $\pi/2$ from ϕ and therefore subtracts $\pi/(2\omega)$ from the phase shift ψ .