

9.3.9 I have $a = 2$, $B = 45^\circ$, and $c = 4$. I apply the Law of Cosines with B as the favoured angle:

$$a^2 - 2ac \cos B + c^2 = b^2;$$

$$b = \sqrt{a^2 - 2ac \cos B + c^2} = \sqrt{(2)^2 - 2(2)(4) \cos(45^\circ) + (4)^2} = \sqrt{4 - 16 \frac{\sqrt{2}}{2} + 16} = 2\sqrt{5 - 2\sqrt{2}} \approx 2.95.$$

I could now use the Law of Sines to find another angle, but it's surer to use the Law of Cosines again:

$$a^2 - 2ab \cos C + b^2 = c^2;$$

$$C = \arccos \frac{a^2 - c^2 + b^2}{2ab} = \arccos \frac{(2)^2 - (4)^2 + (2\sqrt{5 - 2\sqrt{2}})^2}{2(2)(2\sqrt{5 - 2\sqrt{2}})} = \arccos \frac{\sqrt{5 - 2\sqrt{2}} - \sqrt{10 - 4\sqrt{2}}}{5 - 2\sqrt{2}} \approx 106.3^\circ.$$

I could now find the last angle by subtracting from 180° , but I'll use the Law of Cosines once more, as a check:

$$b^2 - 2bc \cos A + c^2 = a^2;$$

$$A = \arccos \frac{b^2 - a^2 + c^2}{2bc} = \arccos \frac{(2\sqrt{5 - 2\sqrt{2}})^2 - (2)^2 + (4)^2}{2(2\sqrt{5 - 2\sqrt{2}})(4)} = \arccos \frac{4\sqrt{5 - 2\sqrt{2}} - \sqrt{10 - 4\sqrt{2}}}{10 - 4\sqrt{2}} \approx 28.7^\circ.$$

Then as a check, $28.7^\circ + 45^\circ + 106.3^\circ = 180^\circ$, as expected.

9.3.15 I have $a = 9$, $b = 6$, and $c = 4$. I apply the Law of Cosines with any one angle to begin with:

$$a^2 - 2ab \cos C + b^2 = c^2;$$

$$C = \arccos \frac{a^2 - c^2 + b^2}{2ab} = \arccos \frac{(9)^2 - (4)^2 + (6)^2}{2(9)(6)} = \arccos \frac{101}{108} \approx 20.7^\circ.$$

I could now use the Law of Sines to find another angle, but it's surer to use the Law of Cosines again:

$$a^2 - 2ac \cos B + c^2 = b^2;$$

$$B = \arccos \frac{a^2 - b^2 + c^2}{2ac} = \arccos \frac{(9)^2 - (6)^2 + (4)^2}{2(9)(4)} = \arccos \frac{61}{72} \approx 32.1^\circ.$$

I could now find the last angle by subtracting from 180° , but I'll use the Law of Cosines once more, as a check:

$$b^2 - 2bc \cos A + c^2 = a^2;$$

$$A = \arccos \frac{b^2 - a^2 + c^2}{2bc} = \arccos \frac{(6)^2 - (9)^2 + (4)^2}{2(6)(4)} = \arccos \left(-\frac{29}{48} \right) \approx 127.2^\circ.$$

Then as a check, $127.2^\circ + 32.1^\circ + 20.7^\circ = 180^\circ$, as expected.