

The most general form for a sinusoidal function is

$$f(x) = A \sin(\omega x - \phi) + B,$$

where A , B , ω , and ϕ are constant real numbers; in principle, A and ω should be positive. A graph of $y = f(x)$ is on the back page; go look at it. ... OK! that's a pretty complicated deal. So let's break it down bit by bit.

The four parameters

The number B is the **mean** of the function. It is the centre of the output values around which the function oscillates. Look at the graph on the back page and draw (dashed) the line $y = B$. Notice how it runs right through the middle of the graph. This is a characteristic feature of the mean.

Next, A is the **amplitude** of the function. It indicates how far the output values vary from the mean. The highest value that y will reach is $B + A$, and the lowest value that y will reach is $B - A$. Look at the graph and draw the lines $y = B + A$ and $y = B - A$. Notice how these are the highest and lowest that the graph reaches.

Next, ϕ is the **angular phase shift** of the function. We use it to calculate the **phase shift** ψ . The phase shift is given by the formula $\psi = \phi/\omega$. It indicates at what input argument the first cycle starts. If you calculate $f(\psi)$, you'll get B . So the output is the mean whenever the input is the phase shift. The cycle starts when the x -value is the phase shift ψ , and the y -value when the cycle starts is the mean B . Warning: There is a minus sign in front of ϕ in the formula. So if you see a plus sign instead, then this indicates that ϕ is negative (and hence so is ψ , assuming that ω is positive).

Finally, ω is the **angular frequency** of the function. We use it to calculate the **period** T . The period is given by the formula $T = 2\pi/\omega$. This is how much the input changes, in either direction, before the function starts repeating. Since the first cycle begins at $x = \psi$, then it must end at $x = \psi + T$. Look at the graph, and examine the points where the x -values are ψ , $\psi + T$, and $\psi - T$. Notice that the y -value is the mean B in each case.

Steps to drawing the graph

Draw some dashed lines at $y = B$, $y = B + A$, and $y = B - A$. This is sort of a layout for your graph. The line $y = B$ will run right down the middle of the graph, the line $y = B + A$ will run along the top of the graph, and the line $y = B - A$ will run along the bottom of the graph.

Now find the x -value where the first cycle starts; this is the phase shift ψ . Find the x -value where the first cycle ends; you get this from the phase shift and the period as $\psi + T$. The y -values at these points will both be the mean B . So mark the points (ψ, B) and $(\psi + T, B)$.

The cycle begins by going up and to the right, then down, then back up to end. You know how high it must go, up to $B + A$, and how low it must go, down to $B - A$. You know where it starts, at (ψ, B) , and where it ends, at $(\psi + T, B)$. So draw it in.

Now you can copy this cycle to the left, where it will start at $(\psi - T, B)$. And you can copy it to the right, where it will end at $(\psi + 2T, B)$. Just repeat it off to the left and off to the right as far as you like. I drew the graph on the back using these steps.

Variations

I said above that A and ω should be positive. If you have a formula where they appear to be negative, you can change it to a formula where they're positive using these identities:

$$-\sin \theta = \sin(-\theta) = \sin(\theta + \pi).$$

Thus, if A is negative, then this effectively subtracts π from ϕ and therefore subtracts π/ω from the phase shift ψ ; if ω is negative, then the result is similar.

If you have a formula with a cosine instead of a sine, then use this identity:

$$\cos \theta = \sin \left(\theta + \frac{\pi}{2} \right).$$

Thus, using the cosine instead of the sine effectively subtracts $\pi/2$ from ϕ and therefore subtracts $\pi/(2\omega)$ from the phase shift ψ .

Getting the formula from the graph

Assuming that A and ω (and hence T) are positive, there's only one value that each of these could take, given the graph. It's easiest to identify the maximum and minimum value; then the average of these is B , and the difference between either of these and B is A . Also, the distance between successive maxima or minima is T ; then $\omega = 2\pi/T$.

The phase shift is somewhat arbitrary. Generally, we want the phase shift to be as small as possible but positive. So find the first place on the graph to the right of the y -axis where the y -value is B and the graph is increasing. The x -value at that point is ψ . As a check, the angular phase shift $\phi = \omega\psi$ should satisfy $0 \leq \phi < 2\pi$ if you pick ψ according to this procedure.

