Inverse trigonometric operations

You only need one inverse trigonometric operation, say $\sin^{-1}$:

- $\cos^{-1} x = \dfrac{\pi}{2} - \sin^{-1} x$;
- $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{x^2+1}}\right)$;
- $\cot^{-1} x = \dfrac{\pi}{2} - \tan^{-1} x = \dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{x}{\sqrt{x^2+1}}\right)$;
- $\sec^{-1} x = \cos^{-1}\left(\dfrac{1}{x}\right) = \dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{1}{x}\right)$;
- $\csc^{-1} x = \sin^{-1}\left(\dfrac{1}{x}\right)$.

Another important fact is that
$$\cos(\sin^{-1} x) = \sqrt{1-x^2};$$
using this (and $\sin(\sin^{-1} x) = x$), you can get an expression for any trigonometric operation applied to an inverse sine. Combined with the rules in the earlier list above, along with the cofunction identities ($\cos(\pi/2 - \theta) = \sin\theta$ etc), you can get an expression for any trigonometric operation applied to any inverse trigonometric operation.

For example, what is $\cot(\sec^{-1} x)$? It is

$$\cot(\sec^{-1} x) = \cot\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{x}\right)\right) = \tan\left(\sin^{-1}\left(\frac{1}{x}\right)\right) = \frac{\sin\left(\sin^{-1}\left(\frac{1}{x}\right)\right)}{\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)}$$
$$= \frac{1/x}{\sqrt{1-\left(\frac{1}{x}\right)^2}} = \frac{1/x}{\sqrt{\frac{x^2-1}{x^2}}} = \frac{1/x}{\sqrt{x^2-1}/|x|} = \frac{|x|}{x\sqrt{x^2-1}} = \frac{|x|\sqrt{x^2-1}}{x(x^2-1)}.$$

(If $x > 0$, then this simplifies to $\sqrt{x^2-1}/(x^2-1)$.) This is about as complicated as it can get.