Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 Suppose that x is a variable quantity and suppose that $y = 3x$.
a. What is $\Delta_4^5(x^2)$?
b. What is $\Delta_{x=-1}^{x=4}(xy)$?
c. What is $\Delta_7^9(y^2)$?
d. What is $\Delta_5^3(x+9)$?

2 For each of the following expressions, find its antidifferentials (indefinite integrals).
a. $3x^2\,\mathrm{d}x$
b. $(5x^3 - 3x + 4)\,\mathrm{d}x$
c. $\left(\sqrt{y} + \dfrac{1}{y^2}\right)\mathrm{d}y$
d. $\left(2x + \dfrac{2}{x}\right)\mathrm{d}x$

3 Evaluate each of the following definite integrals.
a. $\displaystyle\int_0^5 3x^2\,\mathrm{d}x$
b. $\displaystyle\int_{-1}^3 (5x^3 - 3x + 4)\,\mathrm{d}x$
c. $\displaystyle\int_1^2 \left(\sqrt{y} + \dfrac{1}{y^2}\right)\mathrm{d}y$
d. $\displaystyle\int_4^3 \left(2x + \dfrac{2}{x}\right)\mathrm{d}x$

Due Problems

These problems are due November 17 Thursday.

1 For each of the following expressions, find its antidifferentials (indefinite integrals). Show at least one intermediate step for each.
a. $(2x^3 + 4x)\,\mathrm{d}x$
b. $\left(\dfrac{2}{x^3} + \sqrt[3]{x} + \dfrac{2}{x}\right)\mathrm{d}x$

2 Evaluate each of the following definite integrals. Show at least one intermediate step for each.

a. $\int_3^4 (2x^3 + 4x)\, \mathrm{d}x$

b. $\int_{-8}^{-1} \left(\dfrac{2}{x^3} + \sqrt[3]{x} + \dfrac{2}{x} \right) \mathrm{d}x$

Answers to Practice Problems

Here are the answers to the Practice Problems from the beginning of the assignment.

1

a. Just plug in and subtract:
$$\Delta_4^5(x^2) = \left((5)^2\right) - \left((4)^2\right) = 25 - 16 = 9.$$

b. Since $y = 3x$,
$$\Delta_{x=-1}^{x=4}(xy) = \Delta_{x=-1}^{x=4}\left(x(3x)\right) = \Delta_{-1}^{4}(3x^2) = \left(3(4)^2\right) - \left(3(-1)^2\right) = 48 - 3 = 45.$$

c. This is not a fair question! It *might* mean
$$\Delta_{y=7}^{y=9}(y^2) = \left((9)^2\right) - \left((7)^2\right) = 81 - 49 = 32,$$

but it *might* mean
$$\Delta_{x=7}^{x=9}(y^2) = \Delta_{x=7}^{x=9}\left((3x)^2\right) = \Delta_{7}^{9}(9x^2) = \left(9(9)^2\right) - \left(9(7)^2\right) = 729 - 441 = 288.$$

I should never give you an unclear problem like this one!

d. This is also kind of a trick question, but it is fair, and I might give you a problem like it.
$$\Delta_5^3(x + 9) = \left((5) + 9\right) - ((3) + 9) = 14 - 12 = 2.$$

2

a. Running the power rule in reverse, I add 1 to the exponent and divide the coefficient by the new exponent.
$$3x^2 \, \mathrm{d}x = \mathrm{d}\left(\frac{3}{3}x^3\right) = \mathrm{d}(x^3),$$

so
$$\int 3x^2 \, \mathrm{d}x = x^3 + C,$$

where C is an arbitrary constant.

b. Now I run the power rule in reverse for each term.

$$(5x^3 - 3x + 4)\,dx = (5x^3 - 3x^1 + 4x^0)\,dx = d\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + \frac{4}{1}x\right) = d\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\right),$$

so

$$\int(5x^3 - 3x + 4)\,dx = \frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x + C,$$

where C is an arbitrary constant.

c. I rewrite using exponents so that I can reverse the power rule.

$$\left(\sqrt{y} + \frac{1}{y^2}\right)dy = \left(1y^{1/2} + 1y^{-2}\right)dy = d\left(\frac{1}{3/2}y^{3/2} + \frac{1}{-1}y^{-1}\right) = d\left(\frac{2}{3}\sqrt{y^3} - \frac{1}{y}\right),$$

so

$$\int\left(\sqrt{y} + \frac{1}{y^2}\right)dy = \frac{2}{3}\sqrt{y^3} - \frac{1}{y} + C,$$

where C is an arbitrary constant.

d. When the exponent is -1, the antidifferential involves a logarithm.

$$\left(x + \frac{1}{x}\right)dx = (1x^1 + 1x^{-1})\,dx = d\left(\frac{1}{2}x^2 + 1\ln|x|\right) = d\left(\frac{1}{2}x^2 + \ln|x|\right),$$

so

$$\int\left(x + \frac{1}{x}\right)dx = \frac{1}{2}x^2 + \ln|x| + C,$$

where C is an arbitrary constant.

3

a.
$$\int_0^5 3x^2\,dx = \int_0^5 d(x^3) = \Delta_0^5(x^3) = \left((5)^3\right) - \left((0)^3\right) = 125 - 0 = 125.$$

b.

$$\int_{-1}^3 (5x^3 - 3x + 4)\,dx = \int_{-1}^3 d\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\right) = \Delta_{-1}^3\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\right)$$
$$= \left(\frac{5}{4}(3)^4 - \frac{3}{2}(3)^2 + 4(3)\right) - \left(\frac{5}{4}(-1)^4 - \frac{3}{2}(-1)^2 + 4(-1)\right)$$
$$= \frac{399}{4} - \left(-\frac{17}{4}\right) = 104.$$

c.

$$\int_1^2 \left(\sqrt{y} + \frac{1}{y^2}\right) dy = \int_1^2 d\left(\frac{2}{3}\sqrt{y^3} - \frac{1}{y}\right) = \Delta_1^2\left(\frac{2}{3}\sqrt{y^3} - \frac{1}{y}\right)$$

$$= \left(\frac{2}{3}\sqrt{(2)^3} - \frac{1}{(2)}\right) - \left(\frac{2}{3}\sqrt{(1)^3} - \frac{1}{(1)}\right)$$

$$= \left(\frac{4}{3}\sqrt{2} - \frac{1}{2}\right) - \left(-\frac{1}{3}\right) = -\frac{1}{6} + \frac{4}{3}\sqrt{2} \approx 1.72.$$

d.

$$\int_4^3 \left(2x + \frac{2}{x}\right) dx = \int_4^3 d\left(\frac{1}{2}x^2 + \ln|x|\right) = \Delta_4^3\left(\frac{1}{2}x^2 + \ln|x|\right)$$

$$= \left(\frac{1}{2}(4)^2 + \ln|4|\right) - \left(\frac{1}{2}(3)^2 + \ln|3|\right) = 12 - \frac{15}{2} = \frac{9}{2} = 4.5.$$