Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 *Suppose that x is a variable quantity that changes from 1 to 3, and suppose that $u = 3x$.*

a. *What is Δx?*

$\Delta x = 3 - 1 = 2$.

b. *What is $\Delta(x + 5)$?*

$\Delta(x + 5) = (3 + 5) - (1 + 5) = 8 - 6 = 2$.

c. *What is Δu?*

Either
$$\Delta u = \Delta(3x) = 3(3) - 3(1) = 6;$$

or
$$x = 1 \ \Rightarrow \ u = 3(1) = 3,$$
$$x = 3 \ \Rightarrow \ u = 3(3) = 9,$$
$$\Delta u = 9 - 3 = 6.$$

d. *What is $\Delta(u - ux)$?*

Either
$$\Delta(u - ux) = \Delta(3x - 3x^2) = \left[3(3) - 3(3)^2\right] - \left[3(1) - 3(1)^2\right] = -18;$$

or
$$x = 1 \ \Rightarrow \ u = 3(1) = 3,$$
$$x = 3 \ \Rightarrow \ u = 3(3) = 9,$$
$$\Delta(u - ux) = (9 - 9 \cdot 3) - (3 - 3 \cdot 1) = -18.$$

2 *Suppose that x is a smoothly variable quantity.*

a. *Find $\mathrm{d}(x + 3)$ in terms of $\mathrm{d}x$.*

$\mathrm{d}(x + 3) = \mathrm{d}x$, because 3 is constant.

b. *Find $\mathrm{d}(-5x)$ in terms of $\mathrm{d}x$.*

$\mathrm{d}(-5x) = -5\,\mathrm{d}x$, because -5 is constant.

c. *Differentiate $u = 2x - 5$.*

Either
$$\mathrm{d}u = \mathrm{d}(2x - 5) = 2\,\mathrm{d}x$$

or
$$u = 2x - 5,$$
$$\mathrm{d}u = \mathrm{d}(2x - 5),$$
$$\mathrm{d}u = 2\,\mathrm{d}x.$$

d. *If $u = 3x$ and $v = x + 11$, differentiate $u + v$.*

Either
$$\mathrm{d}(u + v) = \mathrm{d}(3x + x + 11) = \mathrm{d}(4x + 11) = 4\,\mathrm{d}x$$

or

$$du = d(3x) = 3\,dx,$$
$$dv = d(x + 11) = dx,$$
$$d(u + v) = du + dv = 3\,dx + dx = 4\,dx.$$

Due Problems

These problems are due October 13 Thursday.

1 *Suppose that the annual profit from making and selling car seats is given by*

$$P = 60\,000\,x - 50\,x^2 - 5000,$$

where P is the profit in thousands of dollars and x is the number of millions of car seats made and sold that year.

a. *If x changes from 100 to 105, then what is ΔP? (Show at least what numerical calculation you make to decide this.)*

$$\Delta P = \Delta_{100}^{105}(60\,000x - 50\,x^2 - 5000) = \left[60\,000(105) - 50(105)^2 - 5000\right] - \left[60\,000(100) - 50(100)^2 - 5000\right]$$
$$= 5\,743\,750 - 5\,495\,000 = 248\,750.$$

b. *If the firm changes from making 80 million car seats in one year to making 85 million car seats in the next year, then what is the change in its annual profit? (Show at least what numerical calculation you make to decide this.)*

This time, x changes from 80 to 85, so

$$\Delta P = \Delta_{80}^{85}(60\,000x - 50\,x^2 - 5000) = \left[60\,000(85) - 50(85)^2 - 5000\right] - \left[60\,000(80) - 50(80)^2 - 5000\right]$$
$$= 4\,733\,750 - 4\,475\,000 = 258\,750.$$

Therefore, the change in annual profit is 258 750 thousand dollars, or \$258,750,000. (In other words, the firm gains 258.75) million dollars by making this change.)

2 *Given $u = x + 1$ and $v = 2x$, differentiate the following:*

a. $A = 2x + 7$

Because 2 and 7 are constants, $dA = 2\,dx$.

b. $B = 9x - 12$

Similarly, $dB = 9\,dx$.

c. $C = 13 - 6x$

In standard order, $C = -6x + 13$, so $dC = -6\,dx$.