Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 *Differentiate (find the differential of) the following expressions:*

a. $3x^2 + 5x - 4$

$$\begin{aligned}\mathrm{d}(3x^2 + 5x - 4) &= \mathrm{d}(3x^2) + \mathrm{d}(5x) - \mathrm{d}(4) = 3\,\mathrm{d}(x^2) + 5\,\mathrm{d}x - 0 \\ &= 3(2x\,\mathrm{d}x) + 5\,\mathrm{d}x = 6x\,\mathrm{d}x + 5\,\mathrm{d}x = (6x + 5)\,\mathrm{d}x\end{aligned}$$

b. $3\sqrt{x} - 5/x$

$$\mathrm{d}(3\sqrt{x} - 5/x) = 3\,\mathrm{d}(\sqrt{x}) - \mathrm{d}(5/x) = 3\frac{\sqrt{x}\,\mathrm{d}x}{2x} - \left(-\frac{5\,\mathrm{d}x}{x^2}\right) = \left(\frac{3\sqrt{x}}{2x} + \frac{5}{x^2}\right)\mathrm{d}x$$

c. $3xy^2 - 2x^2y$

$$\begin{aligned}\mathrm{d}(3xy^2 - 2x^2y) &= 3\,\mathrm{d}(xy^2) - 2\,\mathrm{d}(x^2y) = 3\Big(y^2\,\mathrm{d}x + x\,\mathrm{d}(y^2)\Big) - 2(y\,\mathrm{d}(x^2) + x^2\,\mathrm{d}y) \\ &= 3(y^2\,\mathrm{d}x + 2xy\,\mathrm{d}y) - 2(2xy\,\mathrm{d}x + x^2\,\mathrm{d}y) \\ &= (3y^2 - 4xy)\,\mathrm{d}x + (6xy - 2x^2)\,\mathrm{d}y\end{aligned}$$

d. $\dfrac{x-a}{x+a}$ *if a is a constant*

$$\begin{aligned}\mathrm{d}\left(\frac{x-a}{x+a}\right) &= \frac{(x+a)\,\mathrm{d}(x-a) - (x-a)\,\mathrm{d}(x+a)}{(x+a)^2} = \frac{(x+a)\,\mathrm{d}x - (x-a)\,\mathrm{d}x}{(x+a)^2} \\ &= \frac{x\,\mathrm{d}x + a\,\mathrm{d}x - x\,\mathrm{d}x + a\,\mathrm{d}x}{(x+a)^2} = \frac{2a\,\mathrm{d}x}{(x+a)^2}\end{aligned}$$

2 *Differentiate the following equations:*

a. $y = 5x^3 - 4x^2 + 3x$

$$\begin{aligned}\mathrm{d}y &= \mathrm{d}(5x^3 - 4x^2 + 3x) \\ &= 5\,\mathrm{d}(x^3) - 4\,\mathrm{d}(x^2) + 3\,\mathrm{d}x \\ &= 5(3x^2\,\mathrm{d}x) - 4(2x\,\mathrm{d}x) + 3\,\mathrm{d}x \\ \mathrm{d}y &= (15x^2 - 8x + 3)\,\mathrm{d}x\end{aligned}$$

b. $q = \dfrac{12}{p+5} - 10$

$$\begin{aligned}\mathrm{d}q &= \mathrm{d}\left(\frac{12}{p+5} - 10\right) \\ &= -\frac{12\,\mathrm{d}(p+5)}{(p+5)^2} - 0 \\ \mathrm{d}q &= -\frac{12\,\mathrm{d}p}{(p+5)^2}\end{aligned}$$

c. $x^2 + y^2 = 1$

$$\begin{aligned} \mathrm{d}(x^2 + y^2) &= \mathrm{d}(1) \\ \mathrm{d}(x^2) + \mathrm{d}(y^2) &= 0 \\ 2x\,\mathrm{d}x + 2y\,\mathrm{d}y &= 0 \end{aligned}$$

d. $(x + y)^2 = 1$

$$\begin{aligned} \mathrm{d}\Big((x + y)^2\Big) &= \mathrm{d}(1) \\ 2(x + y)\,\mathrm{d}(x + y) &= 0 \\ (2x + 2y)(\mathrm{d}x + \mathrm{d}y) &= 0 \\ (2x + 2y)\,\mathrm{d}x + (2x + 2y)\,\mathrm{d}y &= 0 \end{aligned}$$

3 *Find the derivative (sensitivity) of y with respect to x:*

a. $y = 5x^3 - 4x^2 + 3x$

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}(5x^3 - 4x^2 + 3x) \\ \mathrm{d}y &= (15x^2 - 8x + 3)\,\mathrm{d}x \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= 15x^2 - 8x + 3 \end{aligned}$$

b. $y = \dfrac{12}{x + 5} - 10$

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}\left(\frac{12}{x + 5} - 10\right) \\ \mathrm{d}y &= -\frac{12\,\mathrm{d}x}{(x + 5)^2} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{12}{(x + 5)^2} \end{aligned}$$

c. $x^2 + y^2 = 1$

$$\begin{aligned} \mathrm{d}(x^2 + y^2) &= \mathrm{d}(1) \\ 2x\,\mathrm{d}x + 2y\,\mathrm{d}y &= 0 \\ 2y\,\mathrm{d}y &= -2x\,\mathrm{d}x \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x}{2y} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{x}{y} \end{aligned}$$

d. $(x + y)^2 = 1$

$$\begin{aligned} \mathrm{d}\Big((x + y)^2\Big) &= \mathrm{d}(1) \\ (2x + 2y)\,\mathrm{d}x + (2x + 2y)\,\mathrm{d}y &= 0 \\ (2x + 2y)\,\mathrm{d}y &= -(2x + 2y)\,\mathrm{d}x \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x + 2y}{2x + 2y} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -1 \end{aligned}$$

Due Problems

These problems were due October 18 Tuesday.

1 *Differentiate (find the differential of)*

$$2x^5 - 3/x + \sqrt{4x}.$$

(Show at least one intermediate step.)

$$\begin{aligned}\mathrm{d}\Big(2x^5 - 3/x + \sqrt{4x}\Big) &= 2\,\mathrm{d}(x^5) - \mathrm{d}(3/x) + \mathrm{d}\Big(\sqrt{4x}\Big) = 2(5x^4\,\mathrm{d}x) - \frac{3\,\mathrm{d}x}{x^2} + \frac{\sqrt{4x}\,\mathrm{d}(4x)}{2(4x)} \\ &= 10x^4\,\mathrm{d}x - \frac{3\,\mathrm{d}x}{x^2} + \frac{\sqrt{4x}(4\,\mathrm{d}x)}{8x} = \left(10x^4 - \frac{3}{x^2} + \frac{\sqrt{4x}}{2x}\right)\mathrm{d}x\end{aligned}$$

2 *Suppose that*

$$y = \frac{2x}{y-3}.$$

Differentiate this equation. (Show at least one intermediate step.)

$$\begin{aligned}\mathrm{d}y &= \mathrm{d}\left(\frac{2x}{y-3}\right) \\ &= \frac{(y-3)\,\mathrm{d}(2x) - (2x)\,\mathrm{d}(y-3)}{(y-3)^2} \\ &= \frac{(y-3)(2\,\mathrm{d}x) - (2x)\,\mathrm{d}y}{(y-3)^2} \\ \mathrm{d}y &= \frac{2(y-3)\,\mathrm{d}x - 2x\,\mathrm{d}y}{(y-3)^2}\end{aligned}$$

3 *Suppose that*

$$y = 2x^5 - \frac{2}{x^3}$$

always. Find the derivative (sensitivity) of y with respect to x. (Show at least one intermediate step.)

$$\begin{aligned}\mathrm{d}y &= \mathrm{d}\left(2x^5 - \frac{2}{x^3}\right) \\ &= 2\,\mathrm{d}(x^5) - 2\,\mathrm{d}(x^{-3}) \\ &= 2(5x^4\,\mathrm{d}x) - 2(-3x^{-4}\,\mathrm{d}x) \\ \mathrm{d}y &= (10x^4 + 6x^{-4})\,\mathrm{d}x \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= 10x^4 + \frac{6}{x^4}\end{aligned}$$