Practice Problems

The first few problems are for practice; do as many of them as you need until they're easy, or make up more for yourself along the same lines.

1 *Suppose that x is a variable quantity and suppose that $y = 3x$.*

a. *What is $\Delta_4^5(x^2)$?*

Just plug in and subtract:

$$\Delta_4^5(x^2) = \big((5)^2\big) - \big((4)^2\big) = 25 - 16 = 9.$$

b. *What is $\Delta_{x=-1}^{x=4}(xy)$?*

Since $y = 3x$,

$$\Delta_{x=-1}^{x=4}(xy) = \Delta_{x=-1}^{x=4}\big(x(3x)\big) = \Delta_{-1}^{4}(3x^2) = \big(3(4)^2\big) - \big(3(-1)^2\big) = 48 - 3 = 45.$$

c. *What is $\Delta_7^9(y^2)$?*

This is not a fair question! It *might* mean

$$\Delta_{y=7}^{y=9}(y^2) = \big((9)^2\big) - \big((7)^2\big) = 81 - 49 = 32,$$

but it *might* mean

$$\Delta_{x=7}^{x=9}(y^2) = \Delta_{x=7}^{x=9}\big((3x)^2\big) = \Delta_7^9(9x^2) = \big(9(9)^2\big) - \big(9(7)^2\big) = 729 - 441 = 288.$$

I should never give you an unclear problem like this one!

d. *What is $\Delta_5^3(x + 9)$?*

This is also kind of a trick question, but it is fair, and I might give you a problem like it.

$$\Delta_5^3(x + 9) = \big((5) + 9\big) - ((3) + 9) = 14 - 12 = 2.$$

2 *For each of the following expressions, find its antidifferentials (indefinite integrals).*

a. $3x^2\,\mathrm{d}x$

Running the power rule in reverse, I add 1 to the exponent and divide the coefficient by the new exponent.

$$3x^2\,\mathrm{d}x = \mathrm{d}\left(\frac{3}{3}x^3\right) = \mathrm{d}(x^3),$$

so

$$\int 3x^2\,\mathrm{d}x = x^3 + C,$$

where C is an arbitrary constant.

b. $(5x^3 - 3x + 4)\,\mathrm{d}x$

Now I run the power rule in reverse for each term.

$$(5x^3 - 3x + 4)\,\mathrm{d}x = (5x^3 - 3x^1 + 4x^0)\,\mathrm{d}x = \mathrm{d}\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + \frac{4}{1}x\right) = \mathrm{d}\left(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\right),$$

so

$$\int (5x^3 - 3x + 4)\,\mathrm{d}x = \frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x + C,$$

where C is an arbitrary constant.

c. $\left(\sqrt{y}+\frac{1}{y^2}\right)\mathrm{d}y$
I rewrite using exponents so that I can reverse the power rule.

$$\left(\sqrt{y}+\frac{1}{y^2}\right)\mathrm{d}y = \left(1y^{1/2}+1y^{-2}\right)\mathrm{d}y = \mathrm{d}\left(\frac{1}{3/2}y^{3/2}+\frac{1}{-1}y^{-1}\right) = \mathrm{d}\left(\frac{2}{3}\sqrt{y^3}-\frac{1}{y}\right),$$

so

$$\int\left(\sqrt{y}+\frac{1}{y^2}\right)\mathrm{d}y = \frac{2}{3}\sqrt{y^3}-\frac{1}{y}+C,$$

where C is an arbitrary constant.

d. $\left(2x+\frac{2}{x}\right)\mathrm{d}x$
When the exponent is -1, the antidifferential involves a logarithm.

$$\left(x+\frac{1}{x}\right)\mathrm{d}x = (1x^1+1x^{-1})\,\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}x^2+1\ln|x|\right) = \mathrm{d}\left(\frac{1}{2}x^2+\ln|x|\right),$$

so

$$\int\left(x+\frac{1}{x}\right)\mathrm{d}x = \frac{1}{2}x^2+\ln|x|+C,$$

where C is an arbitrary constant.

3 *Evaluate each of the following definite integrals.*

a. $\int_0^5 3x^2\,\mathrm{d}x$

$$\int_0^5 3x^2\,\mathrm{d}x = \int_0^5 \mathrm{d}(x^3) = \Delta_0^5(x^3) = \left((5)^3\right)-\left((0)^3\right) = 125-0 = 125.$$

b. $\int_{-1}^3 (5x^3-3x+4)\,\mathrm{d}x$

$$\begin{aligned}\int_{-1}^3 (5x^3-3x+4)\,\mathrm{d}x &= \int_{-1}^3 \mathrm{d}\left(\frac{5}{4}x^4-\frac{3}{2}x^2+4x\right) = \Delta_{-1}^3\left(\frac{5}{4}x^4-\frac{3}{2}x^2+4x\right)\\ &= \left(\frac{5}{4}(3)^4-\frac{3}{2}(3)^2+4(3)\right)-\left(\frac{5}{4}(-1)^4-\frac{3}{2}(-1)^2+4(-1)\right)\\ &= \frac{399}{4}-\left(-\frac{17}{4}\right) = 104.\end{aligned}$$

c. $\int_1^2\left(\sqrt{y}+\frac{1}{y^2}\right)\mathrm{d}y$

$$\begin{aligned}\int_1^2\left(\sqrt{y}+\frac{1}{y^2}\right)\mathrm{d}y &= \int_1^2 \mathrm{d}\left(\frac{2}{3}\sqrt{y^3}-\frac{1}{y}\right) = \Delta_1^2\left(\frac{2}{3}\sqrt{y^3}-\frac{1}{y}\right)\\ &= \left(\frac{2}{3}\sqrt{(2)^3}-\frac{1}{(2)}\right)-\left(\frac{2}{3}\sqrt{(1)^3}-\frac{1}{(1)}\right)\\ &= \left(\frac{4}{3}\sqrt{2}-\frac{1}{2}\right)-\left(-\frac{1}{3}\right) = -\frac{1}{6}+\frac{4}{3}\sqrt{2}\approx 1.72.\end{aligned}$$

d. $\int_4^3\left(2x+\frac{2}{x}\right)\mathrm{d}x$

$$\begin{aligned}\int_4^3\left(2x+\frac{2}{x}\right)\mathrm{d}x &= \int_4^3 \mathrm{d}\left(\frac{1}{2}x^2+\ln|x|\right) = \Delta_4^3\left(\frac{1}{2}x^2+\ln|x|\right)\\ &= \left(\frac{1}{2}(4)^2+\ln|4|\right)-\left(\frac{1}{2}(3)^2+\ln|3|\right) = 12-\frac{15}{2} = \frac{9}{2} = 4.5.\end{aligned}$$

Due Problems

The following problems were due May 19 Thursday.

1 *For each of the following expressions, find its antidifferentials (indefinite integrals). Show at least one intermediate step for each.*

a. $(3x^2 + 4x)\,\mathrm{d}x$

$$(3x^2 + 4x)\,\mathrm{d}x = (3x^2 + 4x^1)\,\mathrm{d}x = \mathrm{d}\Big(\frac{3}{3}x^3 + \frac{4}{2}x^2\Big) = \mathrm{d}(x^3 + 2x^2),$$

so

$$\int (3x^2 + 4x)\,\mathrm{d}x = x^3 + 2x^2 + C,$$

where C is an arbitrary constant.

b. $\left(\frac{3}{x^2} + \sqrt{x} + \frac{5}{x}\right)\mathrm{d}x$

$$\begin{aligned}\Big(\frac{3}{x^2} + \sqrt{x} + \frac{5}{x}\Big)\,\mathrm{d}x &= \big(3x^{-2} + 1x^{1/2} + 5x^{-1}\big)\,\mathrm{d}x \\ &= \mathrm{d}\Big(\frac{3}{-1}x^{-1} + \frac{1}{3/2}x^{3/2} + 5\ln|x|\Big) = \mathrm{d}\Big(-\frac{3}{x} + \frac{2x\sqrt{x}}{3} + 5\ln|x|\Big),\end{aligned}$$

so

$$\int \Big(\frac{3}{x^2} + \sqrt{x} + \frac{5}{x}\Big)\,\mathrm{d}x = -\frac{3}{x} + \frac{2x\sqrt{x}}{3} + 5\ln|x| + C,$$

where C is an arbitrary constant.

2 *Evaluate each of the following definite integrals.*

a. $\int_5^6 (3x^2 + 4x)\,\mathrm{d}x$

$$\int_5^6 (3x^2 + 4x)\,\mathrm{d}x = \int_5^6 \mathrm{d}(x^3 + 2x^2) = \Delta_5^6(x^3 + 2x^2) = \big((6)^3 + 2(6)^2\big) - \big((5)^3 + 2(5)^2\big) = 288 - 175 = 113.$$

b. $\int_{-3}^{-2} \left(\frac{3}{x^2} + \sqrt{x} + \frac{5}{x}\right)\mathrm{d}x$

$$\begin{aligned}\int_{-3}^{-2} \Big(\frac{3}{x^2} + \sqrt{x} + \frac{5}{x}\Big)\,\mathrm{d}x &= \int_{-3}^{-2} \mathrm{d}\Big(-\frac{3}{x} + \frac{2x\sqrt{x}}{3} + 5\ln|x|\Big) = \Delta_{-3}^{-2}\Big(-\frac{3}{x} + \frac{2x\sqrt{x}}{3} + 5\ln|x|\Big) \\ &= \Big(-\frac{3}{(-2)} + \frac{2(-2)\sqrt{-2}}{3} + 5\ln|-2|\Big) - \Big(-\frac{3}{(-3)} + \frac{2(-3)\sqrt{-3}}{3} + 5\ln|-3|\Big) \\ &= \Big(\frac{3}{2} - \frac{4\sqrt{2}\mathrm{i}}{3} + 5\ln 2\Big) - \big(1 - 2\sqrt{3}\mathrm{i} + 5\ln 3\big) \\ &= \frac{1}{2} + 5\ln 2 - 5\ln 3 - \frac{4}{3}\sqrt{2}\mathrm{i} - 2\sqrt{3}\mathrm{i} \approx -1.527 - 1.578\mathrm{i}.\end{aligned}$$