

## Practice Problems

The first few problems are for practice; do as many of them as you need until they're easy.

1 Suppose that  $x$  is a variable quantity that changes from 1 to 3, and suppose that  $u = 3x$ .

a. What is  $\Delta x$ ?

$$\Delta x = 3 - 1 = 2.$$

b. What is  $\Delta(x + 5)$ ?

$$\Delta(x + 5) = (3 + 5) - (1 + 5) = 8 - 6 = 2.$$

c. What is  $\Delta u$ ?

$$\Delta u = \Delta(3x) = 3(3) - 3(1) = 9 - 3 = 6.$$

d. What is  $\Delta(u - ux)$ ?

$$\Delta(u - ux) = \Delta(3x - 3x^2) = [3(3) - 3(3)^2] - [3(1) - 3(1)^2] = 9 - 27 - 3 + 3 = -18.$$

2 Suppose that  $x$  is a smoothly variable quantity.

a. Find  $d(x + 3)$  in terms of  $dx$ .

$$d(x + 3) = dx, \text{ because } 3 \text{ is constant.}$$

b. Find  $d(-5x)$  in terms of  $dx$ .

$$d(-5x) = -5 dx, \text{ because } -5 \text{ is constant.}$$

c. Differentiate  $u = 2x - 5$ .

$$du = d(2x - 5) = 2 dx.$$

d. If  $u = 3x$  and  $v = x + 11$ , differentiate  $u + v$ .

$$d(u + v) = d(3x + x + 11) = d(4x + 11) = 4 dx.$$

## Due Problems

The following problems were due April 12 Tuesday.

1 Suppose that the profit from making and selling shoes is given by  $P = 200x - x^2$ , where  $P$  is the annual profit in thousands of dollars and  $x$  is the number of millions of pairs of shoes made per year.

a. If  $x$  changes from 50 to 100, then what is  $\Delta P$ ? (Show at least what numerical calculation you make to decide this.)

$$\begin{aligned} \Delta P &= \Delta_{50}^{100}(200x - x^2) = [200(100) - (100)^2] - [200(50) - (50)^2] \\ &= [20\,000 - 10\,000] - [10\,000 - 2\,500] = 10\,000 - 7\,500 = 2\,500. \end{aligned}$$

b. If the firm changes from making 100 million pairs of shoes per year to making 150 million pairs of shoes per year, then what is the change in its annual profit? (Show at least what numerical calculation you make to decide this.)

This time,  $x$  changes from 100 to 150, so

$$\begin{aligned} \Delta P &= \Delta_{100}^{150}(200x - x^2) = [200(150) - (150)^2] - [200(100) - (100)^2] \\ &= [30\,000 - 22\,500] - [20\,000 - 10\,000] = 7\,500 - 10\,000 = -2\,500. \end{aligned}$$

Therefore, the change in annual profit is  $-2500$  thousand dollars, or  $-\$2,500,000$ . (In other words, the firm loses 2.5 million dollars by making this change.)

**2** Differentiate the following:

a.  $A = 3x + 5$

Because 3 and 5 are constants,  $dA = 3 dx$ .

b.  $B = 5x - 9$

Similarly,  $dB = 5 dx$ .

c.  $C = 8 - 7x$

In standard order,  $C = -7x + 8$ , so  $dC = -7 dx$ .