Practice Problems

The first few problems are for practice; do as many of them as you need until they're easy, or make up more for yourself along the same lines.

1 *Differentiate (find the differential of) the following expressions:*

a. $3x^2 + 5x - 4$

$$\mathrm{d}(3x^2 + 5x - 4) = \mathrm{d}(3x^2) + \mathrm{d}(5x) - \mathrm{d}(4) = 3\,\mathrm{d}(x^2) + 5\,\mathrm{d}x - 0$$
$$= 3(2x\,\mathrm{d}x) + 5\,\mathrm{d}x = 6x\,\mathrm{d}x + 5\,\mathrm{d}x = (6x + 5)\,\mathrm{d}x$$

b. $3\sqrt{x} - 5/x$

$$\mathrm{d}(3\sqrt{x} - 5/x) = 3\,\mathrm{d}(\sqrt{x}) - \mathrm{d}(5/x) = 3\frac{\sqrt{x}\,\mathrm{d}x}{2x} - \left(-\frac{5\,\mathrm{d}x}{x^2}\right) = \left(\frac{3\sqrt{x}}{2x} + \frac{5}{x^2}\right)\mathrm{d}x$$

c. $3xy^2 - 2x^2y$

$$\mathrm{d}(3xy^2 - 2x^2y) = 3\,\mathrm{d}(xy^2) - 2\,\mathrm{d}(x^2y) = 3\big(y^2\,\mathrm{d}x + x\,\mathrm{d}(y^2)\big) - 2(y\,\mathrm{d}(x^2) + x^2\,\mathrm{d}y)$$
$$= 3(y^2\,\mathrm{d}x + 2xy\,\mathrm{d}y) - 2(2xy\,\mathrm{d}x + x^2\,\mathrm{d}y)$$
$$= (3y^2 - 4xy)\,\mathrm{d}x + (6xy - 2x^2)\,\mathrm{d}y$$

d. $\dfrac{x - a}{x + a}$ *if a is a constant*

$$\mathrm{d}\left(\frac{x - a}{x + a}\right) = \frac{(x + a)\,\mathrm{d}(x - a) - (x - a)\,\mathrm{d}(x + a)}{(x + a)^2} = \frac{(x + a)\,\mathrm{d}x - (x - a)\,\mathrm{d}x}{(x + a)^2}$$
$$= \frac{x\,\mathrm{d}x + a\,\mathrm{d}x - x\,\mathrm{d}x + a\,\mathrm{d}x}{(x + a)^2} = \frac{2a\,\mathrm{d}x}{(x + a)^2}$$

2 *Differentiate the following equations:*

a. $y = 5x^3 - 4x^2 + 3x$

$$\mathrm{d}y = \mathrm{d}(5x^3 - 4x^2 + 3x)$$
$$= 5\,\mathrm{d}(x^3) - 4\,\mathrm{d}(x^2) + 3\,\mathrm{d}x$$
$$= 5(3x^2\,\mathrm{d}x) - 4(2x\,\mathrm{d}x) + 3\,\mathrm{d}x$$
$$\mathrm{d}y = (15x^2 - 8x + 3)\,\mathrm{d}x$$

b. $q = \dfrac{12}{p + 5} - 10$

$$\mathrm{d}q = \mathrm{d}\left(\frac{12}{p + 5} - 10\right)$$
$$= -\frac{12\,\mathrm{d}(p + 5)}{(p + 5)^2} - 0$$
$$\mathrm{d}q = -\frac{12\,\mathrm{d}p}{(p + 5)^2}$$

c. $x^2 + y^2 = 1$

$$d(x^2 + y^2) = d(1)$$
$$d(x^2) + d(y^2) = 0$$
$$2x\,dx + 2y\,dy = 0$$

d. $(x+y)^2 = 1$

$$d\big((x+y)^2\big) = d(1)$$
$$2(x+y)\,d(x+y) = 0$$
$$(2x+2y)(dx+dy) = 0$$
$$(2x+2y)\,dx + (2x+2y)\,dy = 0$$

3 *Find the derivative (sensitivity) of y with respect to x:*

a. $y = 5x^3 - 4x^2 + 3x$

$$dy = d(5x^3 - 4x^2 + 3x)$$
$$dy = (15x^2 - 8x + 3)\,dx$$
$$\frac{dy}{dx} = 15x^2 - 8x + 3$$

b. $y = \dfrac{12}{x+5} - 10$

$$dy = d\left(\frac{12}{x+5} - 10\right)$$
$$dy = -\frac{12\,dx}{(x+5)^2}$$
$$\frac{dy}{dx} = -\frac{12}{(x+5)^2}$$

c. $x^2 + y^2 = 1$

$$d(x^2 + y^2) = d(1)$$
$$2x\,dx + 2y\,dy = 0$$
$$2y\,dy = -2x\,dx$$
$$\frac{dy}{dx} = -\frac{2x}{2y}$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

d. $(x+y)^2 = 1$

$$d\big((x+y)^2\big) = d(1)$$
$$(2x+2y)\,dx + (2x+2y)\,dy = 0$$
$$(2x+2y)\,dy = -(2x+2y)\,dx$$
$$\frac{dy}{dx} = -\frac{2x+2y}{2x+2y}$$
$$\frac{dy}{dx} = -1$$

Due Problems

The following problems were due April 14 Thursday.

1 *Differentiate (find the differential of)*

$$5x^2 + \sqrt{3x} - 4/x.$$

(Show at least one intermediate step.)

$$\mathrm{d}\left(5x^2 + \sqrt{3x} - \frac{4}{x}\right) = \mathrm{d}(5x^2) + \mathrm{d}(\sqrt{3x}) - \mathrm{d}\left(\frac{4}{x}\right) = 5\,\mathrm{d}(x^2) + \frac{\sqrt{3x}\,\mathrm{d}(3x)}{2(3x)} - \left(-\frac{4\,\mathrm{d}x}{x^2}\right)$$

$$= 10x\,\mathrm{d}x + \frac{\sqrt{3x}(3\,\mathrm{d}x)}{6x} + \frac{4\,\mathrm{d}x}{x^2} = \left(10x + \frac{\sqrt{3x}}{2x} + \frac{4}{x^2}\right)\mathrm{d}x.$$

2 *Suppose that*

$$y = \frac{3x}{y-2}.$$

Differentiate this equation. (Show at least one intermediate step.)

$$\mathrm{d}y = \mathrm{d}\left(\frac{3x}{y-2}\right)$$

$$= \frac{(y-2)\,\mathrm{d}(3x) - 3x\,\mathrm{d}(y-2)}{(y-2)^2}$$

$$= \frac{(y-2)(3\,\mathrm{d}x) - 3x\,(\mathrm{d}y - 0)}{(y-2)^2};$$

$$\mathrm{d}y = \frac{3(y-2)\,\mathrm{d}x - 3x\,\mathrm{d}y}{(y-2)^2}.$$

3 *Suppose that*

$$y = 5x^2 - \frac{3}{x^2}$$

always. Find the derivative (sensitivity) of y with respect to x. (Show at least one intermediate step.)

$$\mathrm{d}y = \mathrm{d}\left(5x^2 - \frac{3}{x^2}\right)$$

$$= \mathrm{d}(5x^2) - \mathrm{d}\left(\frac{3}{x^2}\right)$$

$$= 5\,\mathrm{d}(x^2) + \frac{3\,\mathrm{d}(x^2)}{(x^2)^2}$$

$$= 5(2x\,\mathrm{d}x) + \frac{3(2x\,\mathrm{d}x)}{x^4};$$

$$\mathrm{d}y = \left(10x + \frac{6}{x^3}\right)\mathrm{d}x;$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x + \frac{6}{x^3}.$$