Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 Suppose that x is a variable quantity and suppose that $y = 3x$.

a. What is $\Delta_4^5(x^2)$?

b. What is $\Delta_{x=-1}^{x=4}(xy)$?

c. What is $\Delta_7^9(y^2)$?

d. What is $\Delta_5^3(x+9)$?

2 For each of the following expressions, find its antidifferentials (indefinite integrals).

a. $3x^2\,\mathrm{d}x$

b. $(5x^3 - 3x + 4)\,\mathrm{d}x$

c. $\left(\sqrt{y} + \frac{1}{y^2}\right)\mathrm{d}y$

d. $\left(2x + \frac{2}{x}\right)\mathrm{d}x$

3 Evaluate each of the following definite integrals.

a. $\int_0^5 3x^2\,\mathrm{d}x$

b. $\int_{-1}^3 (5x^3 - 3x + 4)\,\mathrm{d}x$

c. $\int_1^2 \left(\sqrt{y} + \frac{1}{y^2}\right)\mathrm{d}y$

d. $\int_4^3 \left(2x + \frac{2}{x}\right)\mathrm{d}x$

Due Problems

These problems are due November 15 Thursday.

1 For each of the following expressions, find its antidifferentials (indefinite integrals). Show at least one intermediate step for each.

a. $(2x^3 + 4x)\,\mathrm{d}x$

b. $\left(\frac{2}{x^3} + \sqrt[3]{x} + \frac{2}{x}\right)\mathrm{d}x$

2 Using your answers to Problem 1 above, evaluate each of the following definite integrals. Show at least one additional intermediate step for each.

a. $\int_3^4 (2x^3 + 4x)\,\mathrm{d}x$

b. $\int_{-8}^{-1} \left(\frac{2}{x^3} + \sqrt[3]{x} + \frac{2}{x}\right)\mathrm{d}x$

Answers to Practice Problems

Here are the answers to the Practice Problems from the beginning of the assignment.

1

a. Just plug in and subtract:

$$\Delta_4^5(x^2) = \big((5)^2\big) - \big((4)^2\big) = 25 - 16 = 9.$$

b. Since $y = 3x$,

$$\Delta_{x=-1}^{x=4}(xy) = \Delta_{x=-1}^{x=4}\big(x(3x)\big) = \Delta_{-1}^{4}(3x^2) = \big(3(4)^2\big) - \big(3(-1)^2\big) = 48 - 3 = 45.$$

c. This is not a fair question! It *might* mean

$$\Delta_{y=7}^{y=9}(y^2) = \big((9)^2\big) - \big((7)^2\big) = 81 - 49 = 32,$$

but it *might* mean

$$\Delta_{x=7}^{x=9}(y^2) = \Delta_{x=7}^{x=9}\big((3x)^2\big) = \Delta_7^9(9x^2) = \big(9(9)^2\big) - \big(9(7)^2\big) = 729 - 441 = 288.$$

I should never give you an unclear problem like this one!

d. Since $5 < 3$, this is also kind of a trick question, but it is fair, and I might give you a problem like it.

$$\Delta_5^3(x+9) = \big((5)+9\big) - ((3)+9) = 14 - 12 = 2.$$

2

a. Running the power rule in reverse, I add 1 to the exponent and divide the coefficient by the new exponent.

$$3x^2\,\mathrm{d}x = \mathrm{d}\left(\frac{3}{3}x^3\right) = \mathrm{d}(x^3),$$

so

$$\int 3x^2\,\mathrm{d}x = x^3 + C,$$

where C is an arbitrary constant.

b. Now I run the power rule in reverse for each term.

$$(5x^3 - 3x + 4)\,\mathrm{d}x = (5x^3 - 3x^1 + 4x^0)\,\mathrm{d}x = \mathrm{d}\Big(\frac{5}{4}x^4 - \frac{3}{2}x^2 + \frac{4}{1}x\Big) = \mathrm{d}\Big(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\Big),$$

so

$$\int (5x^3 - 3x + 4)\,\mathrm{d}x = \frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x + C,$$

where C is an arbitrary constant.

c. I rewrite using exponents so that I can reverse the power rule.

$$\Big(\sqrt{y} + \frac{1}{y^2}\Big)\,\mathrm{d}y = \big(1y^{1/2} + 1y^{-2}\big)\,\mathrm{d}y = \mathrm{d}\Big(\frac{1}{3/2}y^{3/2} + \frac{1}{-1}y^{-1}\Big) = \mathrm{d}\Big(\frac{2}{3}y\sqrt{y} - \frac{1}{y}\Big),$$

so

$$\int \Big(\sqrt{y} + \frac{1}{y^2}\Big)\,\mathrm{d}y = \frac{2}{3}y\sqrt{y} - \frac{1}{y} + C,$$

where C is an arbitrary constant.

d. When the exponent is -1, the antidifferential involves a logarithm.

$$\Big(2x + \frac{2}{x}\Big)\,\mathrm{d}x = (2x^1 + 2x^{-1})\,\mathrm{d}x = \mathrm{d}\Big(\frac{2}{2}x^2 + 2\ln|x|\Big) = \mathrm{d}(x^2 + 2\ln|x|),$$

so

$$\int \Big(2x + \frac{2}{x}\Big)\,\mathrm{d}x = x^2 + 2\ln|x| + C,$$

where C is an arbitrary constant.

3

a.

$$\int_0^5 3x^2\,\mathrm{d}x = \int_0^5 \mathrm{d}(x^3) = \Delta_0^5(x^3) = \big((5)^3\big) - \big((0)^3\big) = 125 - 0 = 125.$$

b.

$$\begin{aligned}\int_{-1}^3 (5x^3 - 3x + 4)\,\mathrm{d}x &= \int_{-1}^3 \mathrm{d}\Big(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\Big) = \Delta_{-1}^3\Big(\frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x\Big)\\ &= \Big(\frac{5}{4}(3)^4 - \frac{3}{2}(3)^2 + 4(3)\Big) - \Big(\frac{5}{4}(-1)^4 - \frac{3}{2}(-1)^2 + 4(-1)\Big)\\ &= \frac{399}{4} - \Big(-\frac{17}{4}\Big) = 104.\end{aligned}$$

c.

$$\begin{aligned}\int_1^2 \left(\sqrt{y} + \frac{1}{y^2}\right) \mathrm{d}y &= \int_1^2 \mathrm{d}\left(\frac{2}{3}y\sqrt{y} - \frac{1}{y}\right) = \Delta_1^2\left(\frac{2}{3}y\sqrt{y} - \frac{1}{y}\right) \\ &= \left(\frac{2}{3}(2)\sqrt{(2)} - \frac{1}{(2)}\right) - \left(\frac{2}{3}(1)\sqrt{(1)} - \frac{1}{(1)}\right) \\ &= \left(\frac{4}{3}\sqrt{2} - \frac{1}{2}\right) - \left(-\frac{1}{3}\right) = -\frac{1}{6} + \frac{4}{3}\sqrt{2} \approx 1.72.\end{aligned}$$

d.

$$\begin{aligned}\int_4^3 \left(2x + \frac{2}{x}\right) \mathrm{d}x &= \int_4^3 \mathrm{d}(x^2 + 2\ln|x|) = \Delta_4^3(x^2 + 2\ln|x|) \\ &= \left((3)^2 + 2\ln|3|\right) - \left((4)^2 + 2\ln|4|\right) \\ &= (9 + 2\ln 3) - (16 + 4\ln 2) = -7 - 4\ln 2 + 2\ln 3 \approx -7.58.\end{aligned}$$