

## Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 Suppose that  $x$  is a variable quantity that changes from 1 to 3, and suppose that  $u = 3x$ .

a. What is  $\Delta x$ ?

$$\Delta x = 3 - 1 = 2.$$

b. What is  $\Delta(x + 5)$ ?

$$\Delta(x + 5) = (3 + 5) - (1 + 5) = 8 - 6 = 2.$$

c. What is  $\Delta u$ ?

Either

$$\Delta u = \Delta(3x) = 3(3) - 3(1) = 6;$$

or

$$x = 1 \Rightarrow u = 3(1) = 3,$$

$$x = 3 \Rightarrow u = 3(3) = 9,$$

$$\Delta u = 9 - 3 = 6.$$

d. What is  $\Delta(u - ux)$ ?

Either

$$\Delta(u - ux) = \Delta(3x - 3x^2) = [3(3) - 3(3)^2] - [3(1) - 3(1)^2] = -18;$$

or

$$x = 1 \Rightarrow u = 3(1) = 3,$$

$$x = 3 \Rightarrow u = 3(3) = 9,$$

$$\Delta(u - ux) = (9 - 9 \cdot 3) - (3 - 3 \cdot 1) = -18.$$

2 Suppose that  $x$  is a smoothly variable quantity.

a. Find  $d(x + 3)$  in terms of  $dx$ .

$$d(x + 3) = dx, \text{ because } 3 \text{ is constant.}$$

b. Find  $d(-5x)$  in terms of  $dx$ .

$$d(-5x) = -5 dx, \text{ because } -5 \text{ is constant.}$$

c. Differentiate (find the differential of)  $u = 2x - 5$ .

$$du = d(2x - 5) = d(2x) = 2 dx.$$

d. If  $u = 3x$  and  $v = x + 11$ , differentiate  $u + v$ .

Either

$$d(u + v) = d(3x + x + 11) = d(4x + 11) = 4 dx$$

or

$$du = d(3x) = 3 dx,$$

$$dv = d(x + 11) = dx,$$

$$d(u + v) = du + dv = 3 dx + dx = 4 dx.$$

## Due Problems

These problems are due October 11 Thursday.

- 1 Suppose that the annual profit from making and selling car seats is given by

$$P = 600x - 50x^2 - 500,$$

where  $P$  is the profit in thousands of dollars and  $x$  is the number of millions of car seats made and sold that year.

- a. If  $x$  changes from 4 to 5, then what is  $\Delta P$ ? (Show at least what numerical calculation you make to decide this.)

$$\begin{aligned}\Delta P &= \Delta_4^5(600x - 50x^2) = [600(5) - 50(5)^2] - [600(4) - 50(4)^2] \\ &= 1750 - 1600 = 150.\end{aligned}$$

- b. If the firm changes from making 4 million car seats in one year to making 5 million car seats in the next year, then what is the change in its annual profit? (Give an answer consistent with part a, and be sure to include correct units of measurement.)

As  $x$  changes from 4 to 5, I already know that  $\Delta P = 150$ . Therefore, the change in annual profit is 150 **thousand dollars**.

- 2 Given  $u = x + 1$  and  $v = 2x + 3$ , differentiate (find the differentials of) the following:

- a.  $5x + 7$

Because 5 and 7 are constants,  $d(5x + 7) = 5 dx$ .

- b.  $v$

Because 2 and 3 are constants,  $dv = d(2x + 3) = 2 dx$ .

- c.  $13u - 6v$

Either

$$d(13u - 6v) = d(13(x + 1) - 6(2x + 3)) = d(x - 5) = dx$$

or

$$du = d(x + 1) = dx,$$

$$dv = d(2x + 3) = 2 dx,$$

$$d(13u - 6v) = 13 du - 6 dv = 13(dx) - 6(2 dx) = dx.$$