Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice. The answers to these are at the end of this handout.

1 *Differentiate (find the differential of) the following expressions:*

a. $3x^2 + 5x - 4$

$$\begin{aligned} \mathrm{d}(3x^2 + 5x - 4) &= \mathrm{d}(3x^2) + \mathrm{d}(5x) - \mathrm{d}(4) = 3\,\mathrm{d}(x^2) + 5\,\mathrm{d}x - 0 \\ &= 3(2x\,\mathrm{d}x) + 5\,\mathrm{d}x = 6x\,\mathrm{d}x + 5\,\mathrm{d}x = (6x+5)\,\mathrm{d}x. \end{aligned}$$

b. $3\sqrt{x} - 5/x$

$$\mathrm{d}(3\sqrt{x} - 5/x) = 3\,\mathrm{d}(\sqrt{x}) - \mathrm{d}(5/x) = 3\frac{\sqrt{x}\,\mathrm{d}x}{2x} - \left(-\frac{5\,\mathrm{d}x}{x^2}\right) = \left(\frac{3\sqrt{x}}{2x} + \frac{5}{x^2}\right)\mathrm{d}x.$$

c. $3pq^2 - 2p^2q$

$$\begin{aligned} \mathrm{d}(3pq^2 - 2p^2q) &= 3\,\mathrm{d}(pq^2) - 2\,\mathrm{d}(p^2q) = 3\big(q^2\,\mathrm{d}p + p\,\mathrm{d}(q^2)\big) - 2(q\,\mathrm{d}(p^2) + p^2\,\mathrm{d}q) \\ &= 3(q^2\,\mathrm{d}p + 2pq\,\mathrm{d}q) - 2(2pq\,\mathrm{d}p + p^2\,\mathrm{d}q) \\ &= (3q^2 - 4pq)\,\mathrm{d}p + (6pq - 2p^2)\,\mathrm{d}q. \end{aligned}$$

d. $\dfrac{x-a}{x+a}$ *if* a *is a constant*

$$\begin{aligned} \mathrm{d}\left(\frac{x-a}{x+a}\right) &= \frac{(x+a)\,\mathrm{d}(x-a) - (x-a)\,\mathrm{d}(x+a)}{(x+a)^2} = \frac{(x+a)\,\mathrm{d}x - (x-a)\,\mathrm{d}x}{(x+a)^2} \\ &= \frac{x\,\mathrm{d}x + a\,\mathrm{d}x - x\,\mathrm{d}x + a\,\mathrm{d}x}{(x+a)^2} = \frac{2a\,\mathrm{d}x}{(x+a)^2}. \end{aligned}$$

2 *Differentiate the following equations:*

a. $y = 5x^3 - 4x^2 + 3x$

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}(5x^3 - 4x^2 + 3x); \\ &= 5\,\mathrm{d}(x^3) - 4\,\mathrm{d}(x^2) + 3\,\mathrm{d}x; \\ &= 5(3x^2\,\mathrm{d}x) - 4(2x\,\mathrm{d}x) + 3\,\mathrm{d}x; \\ \mathrm{d}y &= (15x^2 - 8x + 3)\,\mathrm{d}x. \end{aligned}$$

b. $y = \dfrac{12}{x+5} - 10$

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}\Big(\frac{12}{x+5} - 10\Big); \\ &= -\frac{12\,\mathrm{d}(x+5)}{(x+5)^2} - 0; \\ \mathrm{d}y &= -\frac{12\,\mathrm{d}x}{(x+5)^2}. \end{aligned}$$

c. $x^2 + y^2 = 1$

$$\begin{aligned} \mathrm{d}(x^2 + y^2) &= \mathrm{d}(1); \\ \mathrm{d}(x^2) + \mathrm{d}(y^2) &= 0; \\ 2x\,\mathrm{d}x + 2y\,\mathrm{d}y &= 0. \end{aligned}$$

d. $(x+y)^2 = 1$

$$\begin{aligned} \mathrm{d}\big((x+y)^2\big) &= \mathrm{d}(1); \\ 2(x+y)\,\mathrm{d}(x+y) &= 0; \\ (2x+2y)(\mathrm{d}x + \mathrm{d}y) &= 0; \\ (2x+2y)\,\mathrm{d}x + (2x+2y)\,\mathrm{d}y &= 0. \end{aligned}$$

3 *Find the derivative (sensitivity) of y with respect to x:*

a. $y = 5x^3 - 4x^2 + 3x$

Following problem (2.a),

$$\mathrm{d}y = (15x^2 - 8x + 3)\,\mathrm{d}x.$$

Therefore,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 - 8x + 3.$$

b. $y = \dfrac{12}{x+5} - 10$

Following problem (2.b),

$$\mathrm{d}y = -\frac{12\,\mathrm{d}x}{(x+5)^2}.$$

Therefore,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{12}{(x+5)^2}.$$

c. $x^2 + y^2 = 1$

Following problem (2.c),

$$2x\,\mathrm{d}x + 2y\,\mathrm{d}y = 0.$$

Therefore,

$$\begin{aligned} 2y\,\mathrm{d}y &= -2x\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x}{2y}; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{x}{y}. \end{aligned}$$

d. $(x + y)^2 = 1$

Following problem (2.d),

$$(2x + 2y)\,\mathrm{d}x + (2x + 2y)\,\mathrm{d}y = 0.$$

Therefore,

$$\begin{aligned} (2x + 2y)\,\mathrm{d}y &= -(2x + 2y)\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x + 2y}{2x + 2y}; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -1. \end{aligned}$$

Due Problems

These problems are due October 16 Tuesday.

1 *Differentiate (find the differential of)*

$$3x^6 - 4/x + \sqrt[3]{5x}.$$

(Show at least one intermediate step.)

$$\begin{aligned} \mathrm{d}\left(3x^6 - 4/x + \sqrt[3]{5x}\right) &= 3\,\mathrm{d}(x^6) - \mathrm{d}(4/x) + \mathrm{d}\left(\sqrt[3]{5x}\right) = 3(6x^5\,\mathrm{d}x) - \frac{-4\,\mathrm{d}x}{x^2} + \frac{\sqrt[3]{5x}\,\mathrm{d}(5x)}{3(5x)} \\ &= 18x^5\,\mathrm{d}x + \frac{4\,\mathrm{d}x}{x^2} + \frac{\sqrt[3]{5x}(5\,\mathrm{d}x)}{15x} = \left(18x^5 + \frac{4}{x^2} + \frac{\sqrt[3]{5x}}{3x}\right)\mathrm{d}x. \end{aligned}$$

2 *Suppose that*

$$y = \frac{3x}{y-2}.$$

Differentiate this equation. (Show at least one intermediate step.)

$$\begin{aligned}
\mathrm{d}y &= \mathrm{d}\left(\frac{3x}{y-2}\right) \\
&= \frac{(y-2)\,\mathrm{d}(3x) - (3x)\,\mathrm{d}(y-2)}{(y-2)^2} \\
&= \frac{(y-2)(3\,\mathrm{d}x) - (3x)\,\mathrm{d}y}{(y-2)^2} \\
\mathrm{d}y &= \frac{3(y-2)\,\mathrm{d}x - 3x\,\mathrm{d}y}{(y-2)^2}.
\end{aligned}$$

3 *Suppose that*

$$y = 2x^4 - \frac{4}{x^3}$$

always. Find the derivative (sensitivity) of y with respect to x. (Show at least one intermediate step.)

$$\begin{aligned}
\mathrm{d}y &= \mathrm{d}\left(2x^4 - \frac{4}{x^3}\right) \\
&= 2\,\mathrm{d}(x^4) - 4\,\mathrm{d}(x^{-3}) \\
&= 2(4x^3\,\mathrm{d}x) - 4(-3x^{-4}\,\mathrm{d}x) \\
\mathrm{d}y &= (8x^3 + 12x^{-4})\,\mathrm{d}x \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= 8x^3 + \frac{12}{x^4}.
\end{aligned}$$