Practice Problems

These problems are not to be handed in, but try them first; also try the even problems if you need more practice.

- From §3-7 (pages 200–203): 1–19 odd, 31, 35, 37;
- From §4-6 (pages 254&255): 1, 3, 5, 9, 17, 21, 25, 29, 31.

The answers to these should be in the back of your textbook.

Due Problems

These problems are due October 23 Tuesday.

1 *Suppose that research for a small automobile company suggests that the annual revenue from selling x cars per year will be $25\,000\,x - 5x^2$ dollars, while the annual cost of producing x cars per year will be $10\,000 + 5000\,x$ dollars. Suppose that the company made and sold 2000 cars last year.*

Let R be the revenue, and let C be the cost, both in dollars per year. That is,

$$R = 25\,000\,x - 5x^2;$$
$$C = 10\,000 + 5000\,x.$$

Also, x was 2000 last year.

a. *What was the company's marginal revenue last year? (Show at least what numerical calculation you make, as well as your final answer in words.)*

Marginal revenue is the derivative of revenue R with respect to quantity x:

$$R = 25\,000\,x - 5x^2;$$
$$\mathrm{d}R = \mathrm{d}(25\,000\,x - 5x^2) = 25\,000\,\mathrm{d}x - 10x\,\mathrm{d}x;$$
$$\frac{\mathrm{d}R}{\mathrm{d}x} = 25\,000 - 10x.$$

Since $x = 2000$ last year, $\mathrm{d}R/\mathrm{d}x$ was $25\,000 - 10(2000) = 5000$ last year, so the marginal revenue was \$5000 (per car).

b. *What was the company's marginal cost last year? (Show at least what numerical calculation you make, as well as your final answer in words.)*

Marginal cost is the derivative of cost C with respect to quantity x:

$$C = 10\,000 + 5000\,x;$$
$$\mathrm{d}C = \mathrm{d}(10\,000 + 5000\,x) = 5000\,\mathrm{d}x;$$
$$\frac{\mathrm{d}C}{\mathrm{d}x} = 5000.$$

Thus, the marginal revenue was also \$5000 (per car).

c. *What was the company's marginal profit last year? (Show at least what numerical calculation you make, as well as your final answer in words.)*

Profit is revenue minus cost:

$$P = R - C;$$
$$\mathrm{d}P = \mathrm{d}(R - C) = \mathrm{d}R - \mathrm{d}C;$$
$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mathrm{d}R - \mathrm{d}C}{\mathrm{d}x} = \frac{\mathrm{d}R}{\mathrm{d}x} - \frac{\mathrm{d}C}{\mathrm{d}x};$$
$$\frac{\mathrm{d}P}{\mathrm{d}x} = 5000 - 5000 = 0.$$

Thus, the marginal profit was zero.

2 *A patient is given an injection of medication. Suppose that, t hours after the injection, the amount of medication (in cubic centimetres) in the bloodstream of the patient is* $\frac{100}{t^2+1}$.

Let x be the amount of medication after t hours, in cubic centimetres. That is,

$$x = \frac{100}{t^2+1}.$$

a. *How fast is the medication leaving the bloodstream after 1 hour? (Show at least what numerical calculation you make, as well as your final answer in words.)*

The speed at which a quantity changes is its derivative with respect to time. So,

$$\begin{aligned} \mathrm{d}x &= \mathrm{d}\left(\frac{100}{t^2+1}\right) \\ &= -\frac{100\,\mathrm{d}(t^2+1)}{(t^2+1)^2} \\ &= -\frac{100(2t\,\mathrm{d}t)}{(t^2+1)^2}; \\ \frac{\mathrm{d}x}{\mathrm{d}t} &= -\frac{200t}{(t^2+1)^2}. \end{aligned}$$

After 1 hour, $t = 1$, so

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{200(1)}{\left((1)^2+1\right)^2} = -50$$

then. Therefore, the amount of medicine is changing by $-50\,\mathrm{cm}^3/\mathrm{h}$; in other words, it's leaving the bloodstream at a speed of 50 cubic centimetres per hour.

b. *As the medication leaves the bloodstream, it enters the patient's cells. Theoretically, if there is $x\,\mathrm{cm}^3$ of medication left in the patient's bloodstream and the medication has entered y billion cells, then $x + 2y = 100$. If this theory is accurate, then how fast is the medication entering the patient's cells after 1 hour? (Show at least what numerical calculation you make, as well as your final answer in words.)*

First, I differentiate the equation relating x and y. Next, since the problem discusses the rate of change with time, I divide by $\mathrm{d}t$. Finally, I use the known value of $\mathrm{d}x/\mathrm{d}t$ from part (a) to find $\mathrm{d}y/\mathrm{d}t$.

$$\begin{aligned} x + 2y &= 100; \\ \mathrm{d}(x+2y) &= \mathrm{d}(100); \\ \mathrm{d}x + 2\,\mathrm{d}y &= 0; \\ \frac{\mathrm{d}x}{\mathrm{d}t} + 2\frac{\mathrm{d}y}{\mathrm{d}t} &= 0; \\ (-50) + 2\frac{\mathrm{d}y}{\mathrm{d}t} &= 0; \\ 2\frac{\mathrm{d}y}{\mathrm{d}t} &= 50; \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= 25. \end{aligned}$$

Therefore, the number of cells that the medicine has entered is changing by $25 \times 10^9/\mathrm{h}$; in other words, it's entering the cells at a speed of 25 billion per hour.