

1 Given

$$y = 3(x - 4)^2,$$

find the derivative of y with respect to x .

2 Given

$$x = \sqrt{t^2 - 1},$$

find the derivative of x with respect to t .

3 Given

$$5x + 6y = x^5 + y^2,$$

find the derivative of y with respect to x .

4 Given

$$p = e^q \ln q,$$

find the derivative of p with respect to q .

5 Given

$$g(x) = \frac{x + 4}{x - 1},$$

find g' .

6 Given

$$f(x) = 4x^3 + 2x^2,$$

find f'' .

7 Evaluate

$$\lim_{x \rightarrow 4} \left(\frac{x^2 - 16}{x - 4} \right).$$

8 Evaluate

$$\lim_{y \rightarrow -\infty} (y^3 + 5).$$

9 Evaluate

$$\lim_{x \rightarrow 2^+} \left(\frac{x^2 - 2}{x - 2} \right).$$

10 Evaluate

$$\lim_{t \rightarrow \infty} e^{\frac{t+3}{t-3}}.$$

11 Given

$$g(x) = \sqrt{400 - x^2},$$

find the maximum and minimum value of g , if they exist.

12 Given

$$f(x) = x^4 + 20x^3 + 100x^2,$$

sketch a graph of f .

13 Find the value of

$$\int_0^4 (2x^3 + 3x^2 - 5) dx.$$

14 Find the value of

$$\int_1^4 \left(\frac{1}{x+1} + e^{2x} \right) dx.$$

15 Solve

$$\frac{dx}{dt} = 2tx^2$$

for x as a function of t .

16 Given

$$f(0) = 10,$$

solve

$$f'(x) = 3x$$

for f .

17 The population of a certain city is given approximately by

$$C = 3t^2 + 1,$$

where C is the city's population in thousands and t is the time in years since the city was founded. The population of the city's metropolitan area is given approximately by

$$M = C + \frac{1}{8}tC,$$

where M is the metropolitan population in thousands. Five years after the city was founded, how fast (at what rate) is its population growing?

18 Refer to the previous question. Five years after the city was founded, how fast is the population of its metropolitan area growing?

- 19** Suppose that the revenue from selling x specialty items in a year is

$$R = 8x - x^2,$$

while the cost is

$$C = 2x + 5,$$

both measured in thousands of dollars. How many items should be sold in a year to maximise profit?

- 20** Refer to the previous question. How many items should be sold in a year to maximise revenue?
- 21** Refer to the previous questions. How many items should be sold in a year to minimise cost?
- 22** The annual relative growth rate of the world population of humans is estimated to be about 1.1% now, and the population was exactly 7 billion right about the beginning of this year. If the same relative growth rate is maintained, what will the population be at the beginning of 2050?
- 23** Starting at age 30 years, suppose that you deposit \$2000 per year continuously into an IRA that earns 6% continuous annual interest. If you retire at age 65, then how much money will be in the account?

- 24** Suppose that the demand and supply for a product are

$$D(x) = 100 - x^2,$$

$$S(x) = 15x,$$

each giving price in dollars per litre as a function of quantity in thousands of litres per day. Calculate the producer surplus.

- 25** Refer to the previous question. Calculate the consumer surplus.

- 26** Approximate

$$\int_1^5 \frac{1}{x} dx$$

using an upper Riemann sum of 4 terms, rounding to 3 digits. (Do *not* give the actual value $\ln 5$ of the integral; the correct answer to this question is an *approximation* to $\ln 5$ that can be calculated using only the elementary operations of addition, subtraction, multiplication, and division.)

- 27** Use the value of

$$f(x) = \sqrt{x}$$

and its derivative at $x = 4$ to approximate $f(4.3)$. (Do *not* give the actual $\sqrt{4.3}$ of the function; the correct answer to this question is an *approximation* to $\sqrt{4.3}$ that can be calculated using only the elementary operations of addition, subtraction, multiplication, and division.)