Practice Problems

These problems are not to be handed in, but try them first; also try the even problems if you need more practice.

1 Differentiate (find the differential of) the following expressions:

a. $3x^2 + 5x - 4$

b. $3\sqrt{x} - 5/x$

c. $3pq^2 - 2p^2q$

d. $\dfrac{x-a}{x+a}$ if a is a constant

2 Differentiate the following equations:

a. $y = 5x^3 - 4x^2 + 3x$

b. $y = \dfrac{12}{x+5} - 10$

c. $x^2 + y^2 = 1$

d. $(x+y)^2 = 1$

3 Find the derivative (sensitivity) of y with respect to x:

a. $y = 5x^3 - 4x^2 + 3x$

b. $y = \dfrac{12}{x+5} - 10$

c. $x^2 + y^2 = 1$

d. $(x+y)^2 = 1$

Due Problems

These problems are due April 12 Thursday.

1 Differentiate (find the differential of)

$$3x^6 - 4/x + \sqrt[3]{5x}.$$

(Show at least one intermediate step.)

2 Suppose that

$$y = \frac{3x}{y-2}.$$

Differentiate this equation. (Show at least one intermediate step.)

3 Suppose that

$$y = 2x^4 - \frac{4}{x^3}$$

always. Find the derivative (sensitivity) of y with respect to x. (Show at least one intermediate step.)

Answers to Practice Problems

Here are the answers to the Practice Problems from the beginning of the assignment.

1

a.

$$\begin{aligned} \mathrm{d}(3x^2 + 5x - 4) &= \mathrm{d}(3x^2) + \mathrm{d}(5x) - \mathrm{d}(4) = 3\,\mathrm{d}(x^2) + 5\,\mathrm{d}x - 0 \\ &= 3(2x\,\mathrm{d}x) + 5\,\mathrm{d}x = 6x\,\mathrm{d}x + 5\,\mathrm{d}x = (6x+5)\,\mathrm{d}x. \end{aligned}$$

b.

$$\mathrm{d}(3\sqrt{x} - 5/x) = 3\,\mathrm{d}(\sqrt{x}) - \mathrm{d}(5/x) = 3\frac{\sqrt{x}\,\mathrm{d}x}{2x} - \left(-\frac{5\,\mathrm{d}x}{x^2}\right) = \left(\frac{3\sqrt{x}}{2x} + \frac{5}{x^2}\right)\mathrm{d}x.$$

c.

$$\begin{aligned} \mathrm{d}(3pq^2 - 2p^2q) &= 3\,\mathrm{d}(pq^2) - 2\,\mathrm{d}(p^2q) = 3\big(q^2\,\mathrm{d}p + p\,\mathrm{d}(q^2)\big) - 2(q\,\mathrm{d}(p^2) + p^2\,\mathrm{d}q) \\ &= 3(q^2\,\mathrm{d}p + 2pq\,\mathrm{d}q) - 2(2pq\,\mathrm{d}p + p^2\,\mathrm{d}q) \\ &= (3q^2 - 4pq)\,\mathrm{d}p + (6pq - 2p^2)\,\mathrm{d}q. \end{aligned}$$

d.

$$\begin{aligned} \mathrm{d}\left(\frac{x-a}{x+a}\right) &= \frac{(x+a)\,\mathrm{d}(x-a) - (x-a)\,\mathrm{d}(x+a)}{(x+a)^2} = \frac{(x+a)\,\mathrm{d}x - (x-a)\,\mathrm{d}x}{(x+a)^2} \\ &= \frac{x\,\mathrm{d}x + a\,\mathrm{d}x - x\,\mathrm{d}x + a\,\mathrm{d}x}{(x+a)^2} = \frac{2a\,\mathrm{d}x}{(x+a)^2}. \end{aligned}$$

2

a.

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}(5x^3 - 4x^2 + 3x); \\ &= 5\,\mathrm{d}(x^3) - 4\,\mathrm{d}(x^2) + 3\,\mathrm{d}x; \\ &= 5(3x^2\,\mathrm{d}x) - 4(2x\,\mathrm{d}x) + 3\,\mathrm{d}x; \\ \mathrm{d}y &= (15x^2 - 8x + 3)\,\mathrm{d}x. \end{aligned}$$

b.

$$\begin{aligned} \mathrm{d}y &= \mathrm{d}\left(\frac{12}{x+5} - 10\right); \\ &= -\frac{12\,\mathrm{d}(x+5)}{(x+5)^2} - 0; \\ \mathrm{d}y &= -\frac{12\,\mathrm{d}x}{(x+5)^2}. \end{aligned}$$

c.

$$\begin{aligned} \mathrm{d}(x^2+y^2) &= \mathrm{d}(1); \\ \mathrm{d}(x^2)+\mathrm{d}(y^2) &= 0; \\ 2x\,\mathrm{d}x + 2y\,\mathrm{d}y &= 0. \end{aligned}$$

d.

$$\begin{aligned} \mathrm{d}\Big((x+y)^2\Big) &= \mathrm{d}(1); \\ 2(x+y)\,\mathrm{d}(x+y) &= 0; \\ (2x+2y)(\mathrm{d}x+\mathrm{d}y) &= 0; \\ (2x+2y)\,\mathrm{d}x + (2x+2y)\,\mathrm{d}y &= 0. \end{aligned}$$

3

a. Following problem (2.a),

$$\mathrm{d}y = (15x^2 - 8x + 3)\,\mathrm{d}x.$$

Therefore,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 - 8x + 3.$$

b. Following problem (2.b),

$$\mathrm{d}y = -\frac{12\,\mathrm{d}x}{(x+5)^2}.$$

Therefore,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{12}{(x+5)^2}.$$

c. Following problem (2.c),

$$2x\,\mathrm{d}x + 2y\,\mathrm{d}y = 0.$$

Therefore,

$$\begin{aligned} 2y\,\mathrm{d}y &= -2x\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x}{2y}; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{x}{y}. \end{aligned}$$

d. Following problem (2.d),

$$(2x+2y)\,\mathrm{d}x + (2x+2y)\,\mathrm{d}y = 0.$$

Therefore,

$$\begin{aligned} (2x+2y)\,\mathrm{d}y &= -(2x+2y)\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x+2y}{2x+2y}; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -1. \end{aligned}$$