Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 Find the second derivative of y with respect to x:

a. $y = 5x^3 - 4x^2 + 3x$

b. $y = \dfrac{12}{x+5} - 10$

c. $x^2 + y^2 = 1$

d. $(x+y)^2 = 1$

2 Find formulas for f' and f'':

a. $f(x) = 3x^3$

b. $f(t) = \sqrt{t-5}$

c. $f(x) = \dfrac{x^2}{x-1}$

d. $f(p) = 5 - p^2$

Due Problems

These problems are due April 17 Tuesday.

1 Given that

$$y = 7x^4 - 12x^3$$

always, find the first and second derivatives of y with respect to x. (Show at least one intermediate step for each.)

2 Given that

$$f(t) = \sqrt[3]{t} - 5/t$$

always, find the first and second derivatives of the function f. (Show at least one intermediate step for each.)

3 Extra credit: In Physics classes, one learns the formula

$$y = a + bt - \frac{1}{2}gt^2$$

for the height y of a projectile that is released at a height a with upward velocity b, where t is the amount of time after the projectile is released and g is the local downward acceleration of gravity. (Note that a, b, and g are constant in this situation.) Verify that this formula is correct (showing in each case what calculation you must make to verify this), as follows:

a. At the time at which the projectile is released, check that the height really is a. (Hint: $t = 0$ when the projectile is released.)

b. At the time at which the projectile is released, check that the upward velocity really is b. (Hint: Upward velocity is the derivative of height with respect to time.)

c. At *all* times, check that the downward acceleration of the particle really is g. (Hint: Upward acceleration is the second derivative of height with respect to time, and downward acceleration is simply the opposite of upward acceleration.)

Answers to Practice Problems

Here are the answers to the Practice Problems from the beginning of the assignment.

1

a. Following problem (3.a) in Homework 4,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^2 - 8x + 3.$$

Therefore,

$$\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \mathrm{d}(15x^2 - 8x + 3) = 30x\,\mathrm{d}x - 8\,\mathrm{d}x,$$

so

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 y = \frac{\mathrm{d}(\mathrm{d}y/\mathrm{d}x)}{\mathrm{d}x} = 30x - 8.$$

b. Following problem (3.b) in Homework 4,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{12}{(x+5)^2} = -12(x+5)^{-2}.$$

Therefore,

$$\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \mathrm{d}\big(-12(x+5)^{-2}\big) = -12\big(-2(x+5)^{-3}\,\mathrm{d}(x+5)\big) = \frac{24\,\mathrm{d}x}{(x+5)^3},$$

so

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 y = \frac{\mathrm{d}(\mathrm{d}y/\mathrm{d}x)}{\mathrm{d}x} = \frac{24}{(x+5)^3}.$$

c. Following problem (3.c) in Homework 4,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}.$$

Therefore,

$$\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \mathrm{d}\left(-\frac{x}{y}\right) = -\frac{y\,\mathrm{d}x - x\,\mathrm{d}y}{y^2} = \frac{x\,\mathrm{d}y - y\,\mathrm{d}x}{y^2},$$

so

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 y = \frac{\mathrm{d}(\mathrm{d}y/\mathrm{d}x)}{\mathrm{d}x} = \frac{x\,\mathrm{d}y/\mathrm{d}x - y}{y^2} = \frac{x(-x/y) - y}{y^2} = \frac{-x^2 - y^2}{y^3} = -\frac{x^2 + y^2}{y^3}.$$

d. Following problem (3.d) in Homework 4,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -1.$$

Therefore,

$$\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \mathrm{d}(-1) = 0,$$

so

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 y = \frac{\mathrm{d}(\mathrm{d}y/\mathrm{d}x)}{\mathrm{d}x} = 0.$$

2

a. I differentiate the formula for $f(x)$ and divide by $\mathrm{d}x$:

$$\begin{aligned} f(x) &= 3x^3; \\ \mathrm{d}f(x) &= \mathrm{d}(3x^3) = 9x^2\,\mathrm{d}x; \\ f'(x) = \frac{\mathrm{d}f(x)}{\mathrm{d}x} &= 9x^2. \end{aligned}$$

To find f'', I do this again:

$$\begin{aligned} f'(x) &= 9x^2; \\ \mathrm{d}f'(x) &= \mathrm{d}(9x^2) = 18x\,\mathrm{d}x; \\ f''(x) = \frac{\mathrm{d}f'(x)}{\mathrm{d}x} &= 18x. \end{aligned}$$

b. I differentiate the formula for $f(t)$ and divide by $\mathrm{d}t$:

$$\begin{aligned} f(t) &= \sqrt{t-5}; \\ \mathrm{d}f(t) &= \mathrm{d}\big(\sqrt{t-5}\big) = \frac{\sqrt{t-5}\,\mathrm{d}(t-5)}{2(t-5)} = \frac{\sqrt{t-5}\,\mathrm{d}t}{2t-10}; \\ f'(t) = \frac{\mathrm{d}f(t)}{\mathrm{d}t} &= \frac{\sqrt{t-5}}{2t-10}. \end{aligned}$$

To find f'', I do this again:

$$\begin{aligned} f'(t) &= \frac{\sqrt{t-5}}{2t-10}; \\ \mathrm{d}f'(t) &= \mathrm{d}\left(\frac{\sqrt{t-5}}{2t-10}\right) = \frac{(2t-10)\,\mathrm{d}(\sqrt{t-5}) - \sqrt{t-5}\,\mathrm{d}(2t-10)}{(2t-10)^2} \\ &= \frac{\sqrt{t-5}\,\mathrm{d}t - 2\sqrt{t-5}\,\mathrm{d}t}{(2t-10)^2} = -\frac{\sqrt{t-5}\,\mathrm{d}t}{(2t-10)^2}; \\ f''(t) = \frac{\mathrm{d}f'(t)}{\mathrm{d}t} &= -\frac{\sqrt{t-5}}{(2t-10)^2}. \end{aligned}$$

c. I differentiate the formula for $f(x)$ and divide by $\mathrm{d}x$:

$$\begin{aligned} f(x) &= \frac{x^2}{x-1}; \\ \mathrm{d}f(x) &= \mathrm{d}\left(\frac{x^2}{x-1}\right) = \frac{(x-1)\,\mathrm{d}(x^2) - x^2\,\mathrm{d}(x-1)}{(x-1)^2} = \frac{2x(x-1)\,\mathrm{d}x - x^2\,\mathrm{d}x}{(x-1)^2}; \\ f'(x) = \frac{\mathrm{d}f(x)}{\mathrm{d}x} &= \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}. \end{aligned}$$

To find f'', I do this again:

$$\begin{aligned} f'(x) &= \frac{x^2-2x}{(x-1)^2} = (x^2-2x)(x-1)^{-2}; \\ \mathrm{d}f'(x) &= \mathrm{d}\big((x^2-2x)(x-1)^{-2}\big) \\ &= (x-1)^{-2}\,\mathrm{d}(x^2-2x) + (x^2-2x)\,\mathrm{d}\big((x-1)^{-2}\big) \\ &= (x-1)^{-2}(2x\,\mathrm{d}x - 2\,\mathrm{d}x) - 2(x^2-2x)(x-1)^{-3}\,\mathrm{d}x \\ &= \frac{2x\,\mathrm{d}x - 2\,\mathrm{d}x}{(x-1)^2} - \frac{2(x^2-2x)\,\mathrm{d}x}{(x-1)^3}; \\ f''(x) = \frac{\mathrm{d}f'(x)}{\mathrm{d}x} &= \frac{2x-2}{(x-1)^2} - \frac{2(x^2-2x)}{(x-1)^3}. \end{aligned}$$

(This could be simplified further, if you wish.)

d. I differentiate the formula for $f(p)$ and divide by $\mathrm{d}p$:

$$\begin{aligned} f(p) &= 5 - p^2; \\ \mathrm{d}f(p) &= \mathrm{d}(5 - p^2) = -2p\,\mathrm{d}p; \\ f'(p) = \frac{\mathrm{d}f(p)}{\mathrm{d}p} &= -2p. \end{aligned}$$

To find f'', I do this again:

$$\begin{aligned} f'(p) &= -2p; \\ \mathrm{d}f'(x) &= \mathrm{d}(-2p) = -2\,\mathrm{d}p; \\ f''(x) = \frac{\mathrm{d}f'(x)}{\mathrm{d}x} &= -2. \end{aligned}$$