Practice Problems

These problems are not to be handed in, but try them first; do as many of them as you need until they're easy, or make up more along the same lines if you need more practice.

1 Evaluate the following limits:

a. $\lim_{x \to 5} (x^2)$

b. $\lim_{x \to 3^-} \left(\frac{x+3}{x-3}\right)$

c. $\lim_{x \to 3} \left(\frac{x^2-7x+12}{x^2-4x+3}\right)$

d. $\lim_{x \to 5} \left(\frac{x+3}{x-5}\right)$

2 Suppose that $y = 2x + 4$ for all x. Evaluate the following limits:

a. $\lim_{x \to 3} (y - 3x)$

b. $\lim_{x \to -2} \left(\frac{xy}{x^2+3x+2}\right)$

c. $\lim_{x \to \infty} \left(\frac{1}{y^2+4y+5}\right)$

d. $\lim_{x \to -\infty} \left(\frac{y^2+3}{y+3}\right)$

Due Problems

These problems are due April 24 Tuesday.

Evaluate the following limits. For each, show at least what numerical calculation (or calculation with infinities) you make. If you use L'Hôpital's Rule, then also show the relevant differentials (or derivatives).

1 $\lim_{x \to 5^+} \left(\frac{2-3x}{x-5}\right)$

2 $\lim_{t \to 6} \left(\frac{12}{(t-6)^2}\right)$

3 $\lim_{x \to \infty} \left(\frac{x+3}{x^2-9}\right)$

4 **Extra credit:** Look at one of the Practice Problems whose solutions involve L'Hôpital's Rule (1.c, 2.b, and 2.d), and use some clever algebra to solve it *without* using L'Hôpital's Rule. (Hint: Try factoring each side of the fraction.)

Answers to Practice Problems

Here are the answers to the Practice Problems from the beginning of the assignment.

1

a. I'll start by trying to just plug in the indicated value for x:

$$\lim_{x\to 5}\left(x^2\right) = (5)^2 = 25.$$

It works!

b. Again, I'll start by trying to plug in the indicated value for x:

$$\lim_{x\to 3^-}\left(\frac{x+3}{x-3}\right) = \frac{(3)+3}{(3)-3} = \frac{6}{0}.$$

This is a positive number divided by zero, so the answer may be $\pm\infty$. Since $x-3<0$ while $x<3$, the sign of the denominator is consistently opposite to the sign of the numerator, so the limit is $-\infty$:

$$\lim_{x\to 3^-}\left(\frac{x+3}{x-3}\right) = -\infty.$$

c. Again, I'll try plugging in:

$$\lim_{x\to 3}\left(\frac{x^2-7x+12}{x^2-4x+3}\right) = \frac{(3)^2-7(3)+12}{(3)^2-4(3)+3} = \frac{0}{0}.$$

This is an indeterminate form, so I'll try L'Hôpital's Rule and plug in again:

$$\lim_{x\to 3}\left(\frac{x^2-7x+12}{x^2-4x+3}\right) = \lim_{x\to 3}\left(\frac{\mathrm{d}(x^2-7x+12)}{\mathrm{d}(x^2-4x+3)}\right) = \lim_{x\to 3}\left(\frac{(2x-7)\,\mathrm{d}x}{(2x-4)\,\mathrm{d}x}\right)$$
$$= \lim_{x\to 3}\left(\frac{2x-7}{2x-4}\right) = \frac{2(3)-7}{2(3)-4} = -\frac{1}{2}.$$

d. Start by plugging in:

$$\lim_{x\to 5}\left(\frac{x+3}{x-5}\right) = \frac{5+3}{5-5} = \frac{8}{0}.$$

This is a positive number divided by zero, so the answer may be $\pm\infty$. Since $x-5<0$ while $x<5$ but $x-5>0$ while $x>5$, the sign of the denominator is inconsistent. Therefore, the limit is **undefined** for us. You might also write

$$\lim_{x\to 5}\left(\frac{x+3}{x-5}\right) = \pm\infty$$

and leave it at that.

2

a. First,

$$\lim_{x\to 3} y = \lim_{x\to 3}(2x+4) = 2(3)+4 = 10,$$

so I can plug this in for y whenever $x \to 3$. Then

$$\lim_{x\to 3}(y-3x) = 10 - 3(3) = 1.$$

b. First,

$$\lim_{x\to -2} y = \lim_{x\to -2}(2x+4) = 2(-2)+4 = 0.$$

Then

$$\lim_{x\to -2}\left(\frac{xy}{x^2+3x+2}\right) = \frac{(-2)(0)}{(-2)^2+3(-2)+2} = \frac{0}{0}.$$

This is an indeterminate form, so I'll try L'Hôpital's Rule. Since this involves differentials, I'll find $\mathrm{d}y$ first:

$$\mathrm{d}y = \mathrm{d}(2x+4) = 2\,\mathrm{d}x.$$

Now I can apply L'Hôpital and plug in again:

$$\begin{aligned}\lim_{x\to -2}\left(\frac{xy}{x^2+3x+2}\right) &= \lim_{x\to -2}\left(\frac{\mathrm{d}(xy)}{\mathrm{d}(x^2+3x+2)}\right) = \lim_{x\to -2}\left(\frac{y\,\mathrm{d}x + x\,\mathrm{d}y}{2x\,\mathrm{d}x+3\,\mathrm{d}x}\right)\\ &= \lim_{x\to -2}\left(\frac{y\,\mathrm{d}x + x(2\,\mathrm{d}x)}{2x\,\mathrm{d}x+3\,\mathrm{d}x}\right) = \lim_{x\to -2}\left(\frac{y+2x}{2x+3}\right) = \frac{(0)+2(-2)}{2(-2)+3} = 4.\end{aligned}$$

c. I'll plug in the infinite value for x and use the rules for arithmetic with infinity. First,

$$\lim_{x\to\infty} y = \lim_{x\to\infty}(2x+4) = 2(\infty)+4 = \infty+4 = \infty.$$

Then

$$\lim_{x\to\infty}\left(\frac{1}{y^2+4y+5}\right) = \frac{1}{(\infty)^2+4(\infty)+5} = \frac{1}{\infty+\infty+5} = \frac{1}{\infty+5} = \frac{1}{\infty} = 0.$$

d. First,

$$\lim_{x\to -\infty} y = \lim_{x\to -\infty}(2x+4) = 2(-\infty)+4 = -\infty+4 = -\infty.$$

Then

$$\lim_{x\to -\infty}\left(\frac{y^2+3}{y+3}\right) = \frac{(-\infty)^2+3}{-\infty+3} = \frac{\infty^2+3}{-\infty} = -\frac{\infty+3}{\infty} = -\frac{\infty}{\infty}.$$

This is an indeterminate form, so I'll try L'Hôpital's Rule:

$$\begin{aligned}\lim_{x\to -\infty}\left(\frac{y^2+3}{y+3}\right) &= \lim_{x\to -\infty}\left(\frac{\mathrm{d}(y^2+3)}{\mathrm{d}(y+3)}\right) = \lim_{x\to -\infty}\left(\frac{2y\,\mathrm{d}y}{\mathrm{d}y}\right)\\ &= \lim_{x\to -\infty}(2y) = 2(-\infty) = -2\infty = -\infty.\end{aligned}$$

(Since y is the only variable here, there's no need to put $\mathrm{d}y$ in terms of $\mathrm{d}x$.)