5.5.45 Since

$$f(x) = x + x^{-1} + 30x^{-3},$$
$$f'(x) = 1 - x^{-2} - 90x^{-4}.$$

This is always defined for $x \in (0, \infty)$, so set this equal to zero and solve:

$$1 - \frac{1}{x^2} - \frac{90}{x^4} = 0;$$
$$x^4 - x^2 - 90 = 0;$$
$$(x^2 - 10)(x^2 + 9) = 0;$$
$$x^2 - 10 = 0 \text{ or } x^2 + 9 = 0;$$
$$x^2 = 10 \text{ or } x^2 = -9;$$
$$x = \pm\sqrt{10} \text{ or } x = \pm 3\mathrm{i}.$$

For $x \in (0, \infty)$, the only solution is

$$x = \sqrt{10}.$$

Then

$$f\left(\sqrt{10}\right) = \left(\sqrt{10}\right) + \frac{1}{\left(\sqrt{10}\right)} + \frac{30}{\left(\sqrt{10}\right)^3} = \frac{7\sqrt{10}}{5} \approx 4.427.$$

This may or may not actually be the minimum on $(0, \infty)$. To decide this, I need to check some limits:

$$\lim_{x \to 0^+} f(x) = (0) + \frac{1}{(0)} + \frac{30}{(0)^3} = 0 + \frac{1}{0} + \frac{30}{0};$$

since $x > 0$ as $x \to 0^+$, also $x^3 > 0$, so

$$\lim_{x \to 0^+} f(x) = 0 + \infty + \infty = \infty.$$

Also,

$$\lim_{x \to \infty} f(x) = (\infty) + \frac{1}{(\infty)} + \frac{30}{(\infty)^3} = \infty + 0 + 0 = \infty.$$

These are both well greater than 4.427, so the minimum really is

$$\frac{7\sqrt{10}}{5}.$$