The final exam will be on September 22 Tuesday during the normal class period. You may use one sheet of notes that you've written yourself, but not your textbook or anything else not written by you, and you may not communicate with anybody but me. Also, you may use a calculator if you wish, although you shouldn't really need one.

The answers to the questions in this exam are on the last page.

1 Given

$$y = \sqrt{3x^2 + 4},$$

find the derivative of y with respect to x.

a $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{3x^2+4}}{2} = \dfrac{1}{2}(3x^2+4)^{1/2}$

b $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x\sqrt{3x^2+4} = 3x(3x^2+4)^{1/2}$

c $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x\sqrt{3x^2+4}}{3x^2+4} = 3x(3x^2+4)^{-1/2}$

d $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{3x^2+4}}{2(3x^2+4)} = \dfrac{1}{2}(3x^2+4)^{-1/2}$

2 Given

$$3x + 4y = x^2 + y^3,$$

find the derivative of y with respect to x.

a $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2x-3}{3y^2-4}$

b $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3y^2+2x}{7}$

c $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3y^2+2x-3}{4}$

d $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2x-3}{3y-4}$

3 Given

$$x = t\mathrm{e}^{2t},$$

find the derivative of x with respect to t.

a $2\mathrm{e}^{2t}$

b $\mathrm{e}^{2t} + t\mathrm{e}^{2t} = (t+1)\mathrm{e}^{2t}$

c $2t\mathrm{e}^{2t}$

d $\mathrm{e}^{2t} + 2t\mathrm{e}^{2t} = (2t+1)\mathrm{e}^{2t}$

4 Given

$$f(x) = \frac{x+1}{x-4},$$

find f'.

a $f'(x) = -\dfrac{5}{(x+1)^2}$

b $f'(x) = \dfrac{5}{(x-4)^2}$

c $f'(x) = -\dfrac{5}{(x-4)^2}$

d $f'(x) = \dfrac{5}{(x+1)^2}$

5 Given

$$g(x) = 4x^3 + 2x^2,$$

find g''.

a $g''(x) = 12x^2 + 4x$

b $g''(x) = 24x + 4$

c $g''(x) = 24x^2 + 4x$

d $g''(x) = 12x + 4$

6 Evaluate

$$\lim_{x \to 6} \left(\frac{x^2 - 36}{x - 6} \right).$$

a 12

b ∞

c $-\infty$

d undefined

7 Evaluate

$$\lim_{x \to -\infty} (x^4 + 5x^2).$$

a 24

b ∞

c $-\infty$

d undefined

8 Evaluate

$$\lim_{x \to -3^-} \left(\frac{x^2 + 3}{x + 3} \right).$$

a 6

b ∞

c $-\infty$

d undefined

9 Given

$$f(x) = \sqrt{100 - x^3},$$

find the maximum and minimum value of f, if they exist.

a maximum is 10, minimum is 0

b maximum is 10, no minimum

c no maximum, minimum is 0

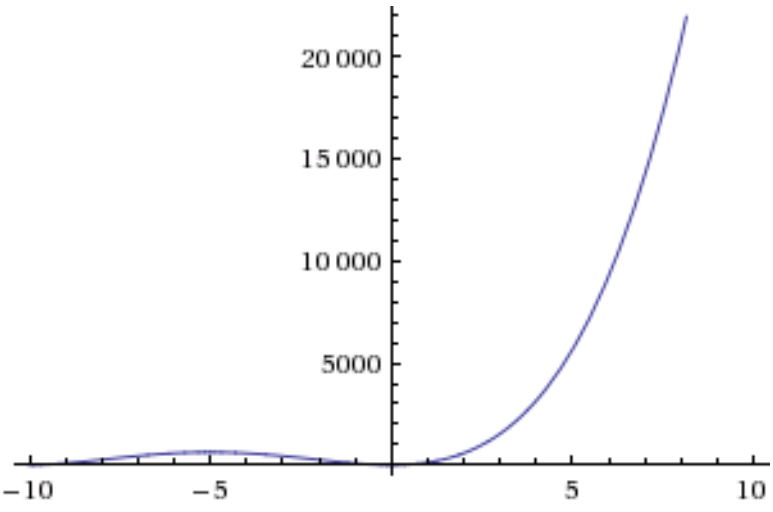
d no maximum, no minimum

10 Given

$$f(x) = x^4 + 20x^3 + 100x^2,$$

sketch a graph of f that shows all intercepts (if any), all local extrema (if any), all asymptotes (if any), and both infinite limits (if applicable).

a



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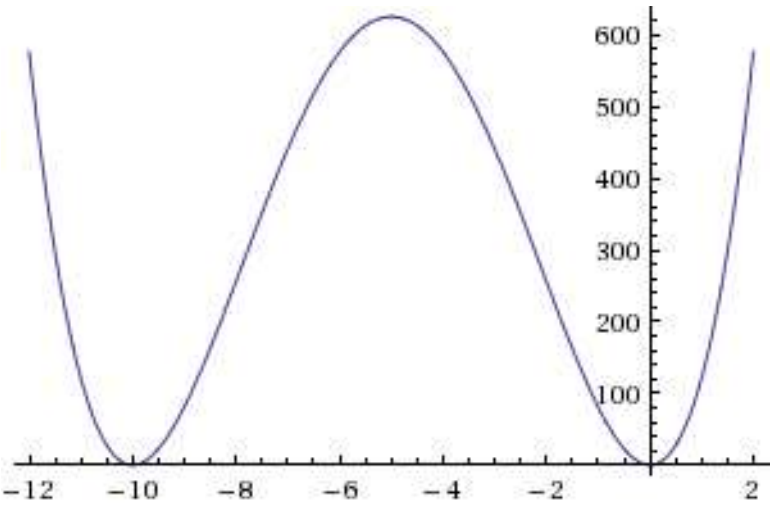
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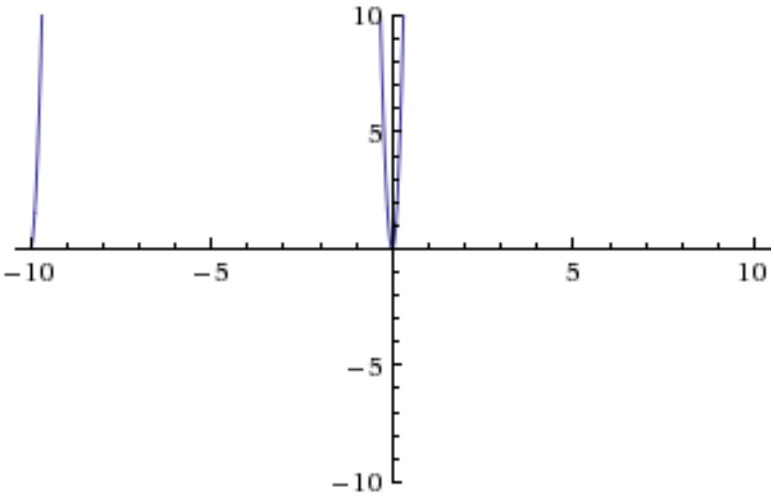
c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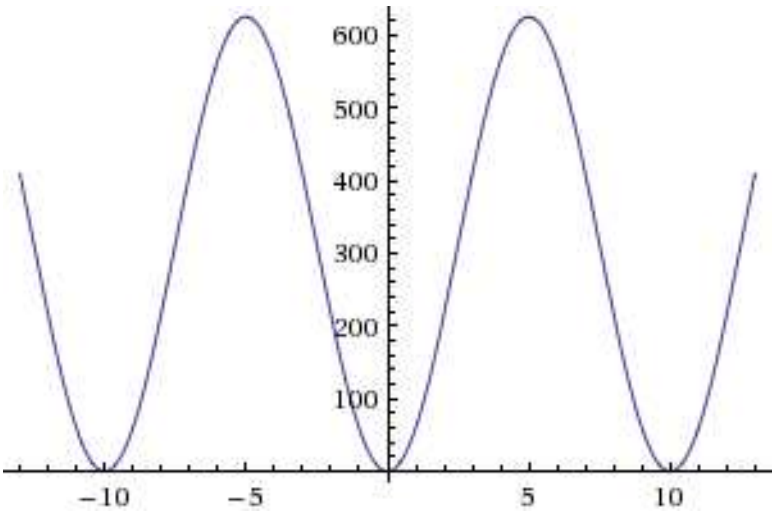
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11 Find the value of

$$\int_0^1 (4x^3 - 3x^2 + 4x - 2)\,\mathrm{d}x.$$

a 4

b 2

c 0

d 3

12 Given

$$f(x) = \frac{1}{x+2} + \mathrm{e}^{3x},$$

find the antiderivatives (indefinite integrals) of f.

a $\int f(x)\,\mathrm{d}x = \frac{1}{2}\ln(x+2) + \frac{1}{3}\mathrm{e}^{3x} + C$

b $\int f(x)\,\mathrm{d}x = \ln(x+2) + \mathrm{e}^{3x} + C$

c $\int f(x)\,\mathrm{d}x = \frac{1}{2}\ln(x+2) + \mathrm{e}^{3x} + C$

d $\int f(x)\,\mathrm{d}x = \ln(x+2) + \frac{1}{3}\mathrm{e}^{3x} + C$

13 Suppose that a leaking oil platform is forming a circular oil slick. At the moment, the radius of this slick is 100 metres, and it's increasing at a rate of 3 metres per hour. How fast is the area of the oil slick increasing?

a $900\pi\,\mathrm{m}^2/\mathrm{h}$

b $30{,}000\pi\,\mathrm{m}^2/\mathrm{h}$

c $300\pi\,\mathrm{m}^2/\mathrm{h}$

d $600\pi\,\mathrm{m}^2/\mathrm{h}$

14 Suppose that the revenue from selling x thousand items in a year is

$$R = 10x - x^2,$$

while the cost to make them is

$$C = 2x + 10,$$

both measured in millions of dollars. How much should be made and sold in a year to maximise profit?

a 2000 per year

b 3000 per year

c 4000 per year

d 5000 per year

15 The annual relative growth rate of the world population of humans is estimated to be about 1.1 % now, and the population was exactly 7 billion right about the beginning of the year 2012. If the same relative growth rate is maintained the whole time, what will the population be at the beginning of 2050?

a $7\mathrm{e}^{209/500} \approx 10.6$ billion

b $7 \cdot 10^{209/500} \approx 18.3$ billion

c $7 \cdot 2^{209/500} \approx 9.3$ billion

d $7 \cdot 7^{209/500} = 7^{709/500} \approx 15.8$ billion

16 The _____ of y is $\mathrm{d}y$.

17 The _____ of f is f'.

18 A(n) _____ of a function is either a minimum or a maximum.

19 A line that a graph approaches arbitrarily closely but does not reach is a(n) _____ of that graph.

20 The expression $\int_a^b f(x)\,\mathrm{d}x$ is a(n) _____ integral.

Answers

1 C, 2 A, 3 D, 4 C, 5 B, 6 A, 7 B, 8 C, 9 A, 10 B, 11 C, 12 D, 13 D, 14 C, 15 A.

16 differential

17 derivative

18 extremum (or extreme value)

19 asymptote

20 definite