4.10.39 Let x be the distance in feet from the pole to the ball's shadow. At all times, we have this proportion from similar triangles:

$$\frac{x}{50} = \frac{x-30}{50-s}.$$

To make things easier, I write this with multiplication instead of with division and simplify:

$$\begin{aligned} x(50-s) &= 50(x-30); \\ 50x - xs &= 50x - 1500; \\ xs &= 1500. \end{aligned}$$

Then I differentiate (take differentials of) both sides:

$$\begin{aligned} \mathrm{d}(xs) &= \mathrm{d}(1500); \\ s\,\mathrm{d}x + x\,\mathrm{d}s &= 0. \end{aligned}$$

Since I'm interested in the speed of change, I divide by $\mathrm{d}t$; I'll also use dot notation to make the algebra easier:

$$\begin{aligned} s\frac{\mathrm{d}x}{\mathrm{d}t} + x\frac{\mathrm{d}s}{\mathrm{d}t} &= 0; \\ s\dot{x} + x\dot{s} &= 0. \end{aligned}$$

Another fact true at all times is given in the problem statement; I'll perform the same steps on it:

$$\begin{aligned} s &= 16t^2; \\ \mathrm{d}s &= \mathrm{d}(16t^2); \\ \mathrm{d}s &= 32t\,\mathrm{d}t; \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= 32t\frac{\mathrm{d}t}{\mathrm{d}t}; \\ \dot{s} &= 32t. \end{aligned}$$

Now I have these four equations in the five quantites $t, s, x, \dot{s}, \dot{x}$:

$$\begin{aligned} s &= 16t^2, \\ xs &= 1500, \\ s\dot{x} + x\dot{s} &= 0, \\ \dot{s} &= 32t. \end{aligned}$$

These are all true in general; I am interested in the moment when $t = 1/2$. Then I can calculate the rest:

$$\begin{aligned} s &= 16t^2 = 16\left(\frac{1}{2}\right)^2 = 4; \\ x &= \frac{1500}{s} = \frac{1500}{4} = 375; \\ \dot{s} &= 32t = 32\left(\frac{1}{2}\right) = 16; \\ \dot{x} &= -\frac{x\dot{s}}{s} = -\frac{(375)(16)}{4} = -1500. \end{aligned}$$

Since we've been measuring distances in metres and times in seconds, the speed at which the shadow of the ball moves is **fifteen hundered metres per second**.