

2.3.11

$$\begin{aligned}\sqrt{3} &< x < \sqrt{5}; \\ \sqrt{3} - 2 &< x - 2 < \sqrt{5} - 2; \\ -(2 - \sqrt{3}) &< x - 2 < \sqrt{5} - 2.\end{aligned}$$

Since $\sqrt{5} - 2 < 2 - \sqrt{3}$ (check on a calculator if you want), use

$$\delta = \sqrt{5} - 2,$$

which is positive.

2.3.23

$$\begin{aligned}|f(x) - L| &< \epsilon; \\ |x^2 - 4| &< 0.5; \\ -0.5 &< x^2 - 4 < 0.5; \\ 3.5 &< x^2 < 4.5.\end{aligned}$$

Since everything here is positive, we can take square roots.

$$\sqrt{3.5} < |x| < \sqrt{4.5}.$$

Since $x \approx c = -2$, we can assume that $|x| = -x$.

$$\begin{aligned}\sqrt{3.5} &< -x < \sqrt{4.5}; \\ -\sqrt{3.5} &> x > -\sqrt{4.5}; \\ -\sqrt{4.5} &< x < -\sqrt{3.5}; \\ 2 - \sqrt{4.5} &< x + 2 < 2 - \sqrt{3.5}; \\ -(\sqrt{4.5} - 2) &< x - (-2) < 2 - \sqrt{3.5}.\end{aligned}$$

Since $\sqrt{4.5} - 2 < 2 - \sqrt{3.5}$, use

$$\delta = \sqrt{4.5} - 2,$$

which is positive.

2.3.39

$$\begin{aligned}|\sqrt{x-5} - 2| &< \epsilon; \\ -\epsilon &< \sqrt{x-5} - 2 < \epsilon; \\ 2 - \epsilon &< \sqrt{x-5} < 2 + \epsilon.\end{aligned}$$

Since $\epsilon \approx 0$, everything here is positive, so we can take squares.

$$\begin{aligned}4 - 4\epsilon + \epsilon^2 &< x - 5 < 4 + 4\epsilon + \epsilon^2; \\ 9 - 4\epsilon + \epsilon^2 &< x < 9 + 4\epsilon + \epsilon^2; \\ -4\epsilon + \epsilon^2 &< x - 9 < 4\epsilon + \epsilon^2; \\ -(4\epsilon - \epsilon^2) &< x - 9 < 4\epsilon + \epsilon^2.\end{aligned}$$

Since $4\epsilon - \epsilon^2 < 4\epsilon + \epsilon^2$, use

$$\delta = 4\epsilon - \epsilon^2,$$

which is positive since $\epsilon \approx 0$.