

2.6.9 As $x \rightarrow \infty$ (so $2x \rightarrow \infty$), $\sin 2x$ tends to no particular value; still, I know something about it, and that's enough:

$$\begin{aligned} -1 &\leq \sin 2x \leq 1; \\ -\frac{1}{x} &\leq \frac{\sin 2x}{x} \leq \frac{1}{x}; \\ \lim_{x \rightarrow \infty} \left(-\frac{1}{x}\right) &\leq \lim_{x \rightarrow \infty} \frac{\sin 2x}{x} \leq \lim_{x \rightarrow \infty} \frac{1}{x}; \\ 0 &\leq \lim_{x \rightarrow \infty} \frac{\sin 2x}{x} \leq 0. \end{aligned}$$

Therefore,

$$\lim_{x \rightarrow \infty} \frac{\sin 2x}{x} = 0.$$

2.6.27 As $x \rightarrow \infty$, both $2\sqrt{x} + x^{-1} \rightarrow \infty$ and $3x - 7 \rightarrow \infty$, so factor out leading powers of x :

$$\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7} = \lim_{x \rightarrow \infty} \frac{\sqrt{x}(2 + x^{-3/2})}{x(3 - 7/x)} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \frac{2 + x^{-3/2}}{3 - 7/x} = 0 \frac{2 + 0}{3 - 0} = 0.$$

2.6.41 As $x \rightarrow -8^+$, $x + 8 \rightarrow 0$, and I can't divide by 0. However, I can still figure out an infinite limit:

$$\begin{aligned} x &\rightarrow -8^+; \\ x &> -8; \\ x + 8 &> 0. \end{aligned}$$

Since $2x \rightarrow 16 > 0$ and $x + 8 > 0$, the result is positive:

$$\lim_{x \rightarrow -8^+} \frac{2x}{x + 8} = \infty.$$