3.3.3 Clearly we want derivatives of s with respect to t, so take differentials using the appropriate rules and divide by $\mathrm{d}t$.

$$\begin{aligned} s &= 5t^3 - 3t^5; \\ \mathrm{d}s &= \mathrm{d}(5t^3 - 3t^5) \\ &= \mathrm{d}(5t^3) - \mathrm{d}(3t^5) \\ &= 5\,\mathrm{d}(t^3) - 3\,\mathrm{d}(t^5) \\ &= 5(3t^{3-1}\,\mathrm{d}t) - 3(5t^{5-1})\,\mathrm{d}t \\ &= (15t^2 - 15t^4)\,\mathrm{d}t; \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= 15t^2 - 15t^4. \end{aligned}$$

This is the first derivative.

For the second derivative, I continue:

$$\begin{aligned} \mathrm{d}s/\mathrm{d}t &= 15t^2 - 15t^4; \\ \mathrm{d}(\mathrm{d}s/\mathrm{d}t) &= \mathrm{d}(15t^2 - 15t^4) \\ &= \mathrm{d}(15t^2) - \mathrm{d}(15t^4) \\ &= 15\,\mathrm{d}(t^2) - 15\,\mathrm{d}(t^4) \\ &= 15(2t^{2-1}\,\mathrm{d}t) - 15(4t^{4-1}\,\mathrm{d}t) \\ &= (30t - 60t^3)\,\mathrm{d}t; \\ (\mathrm{d}/\mathrm{d}s)^2 t = \frac{\mathrm{d}s/\mathrm{d}t}{\mathrm{d}t} &= 30t - 60t^3. \end{aligned}$$

This is the second derivative.

3.3.31 Since y is given as a function of x, the derivative that we want is $\mathrm{d}y/\mathrm{d}x$.

$$\begin{aligned} y &= x^3\mathrm{e}^x; \\ \mathrm{d}y &= \mathrm{d}(x^3\mathrm{e}^x) \\ &= \mathrm{e}^x\,\mathrm{d}(x^3) + x^3\,\mathrm{d}(\mathrm{e}^x) \\ &= \mathrm{e}^x(3x^{3-1}\,\mathrm{d}x) + x^3(\mathrm{e}^x\,\mathrm{d}x) \\ &= (3x^2\mathrm{e}^x + x^3\mathrm{e}^x)\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= x^3\mathrm{e}^x + 3x^2\mathrm{e}^x = x^2(x+3)\mathrm{e}^x. \end{aligned}$$

3.6.21 We have $y = \mathrm{e}^u$, where $u = -5x$. Since I do things in terms of differentials, this is all automatic.

$$\begin{aligned} y &= \mathrm{e}^{-5x}; \\ \mathrm{d}y &= \mathrm{d}(\mathrm{e}^{-5x}) \\ &= \mathrm{e}^{-5x}\,\mathrm{d}(-5x) \\ &= \mathrm{e}^{-5x}(-5\,\mathrm{d}x) \\ &= -5\mathrm{e}^{-5x}\,\mathrm{d}x; \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= -5\mathrm{e}^{-5x}. \end{aligned}$$

3.6.87.e First,

$$\frac{\mathrm{d}\Big(f\big(g(x)\big)\Big)}{\mathrm{d}x} = \frac{\mathrm{d}(f \circ g)(x)}{\mathrm{d}x} = (f \circ g)'(x) = f'\big(g(x)\big)\, g'(x).$$

Alternatively,

$$\frac{\mathrm{d}\Big(f\big(g(x)\big)\Big)}{\mathrm{d}x} = \frac{f'\big(g(x)\big)\, \mathrm{d}(g(x))}{\mathrm{d}x} = \frac{f'\big(g(x)\big)g'(x)\, \mathrm{d}x}{\mathrm{d}x} = f'(g(x))g'(x).$$

Either way, when $x = 2$, then

$$f'\big(g(x)\big)\, g'(x) = f'\big(g(2)\big)\, g'(2) = f'(2)\,(-3) = -3\,(1/3) = -1.$$