The final exam will be on December 17 Thursday during the normal class period. It should take about an hour if you do it carefully. You may use one sheet of notes that you've written yourself, but not your textbook or anything else not written by you, and you may not communicate with anybody but me. Also, you may use a calculator if you wish, although you shouldn't really need one.

The answers to the questions in this exam are on the last page.

1 Evaluate

$$\lim_{x\to -3^-}\left(\frac{x^2+3}{x+3}\right).$$

a. 6

b. ∞

c. $-\infty$

d. undefined

2 Evaluate

$$\lim_{x\to -\infty}(x^4+5x^2).$$

a. 24

b. ∞

c. $-\infty$

d. undefined

3 Evaluate

$$\lim_{x\to -2}\left(\frac{x^2-5x-14}{x^2+5x+6}\right).$$

a. -9

b. ∞

c. $-\infty$

d. 0

4 Evaluate

$$\lim_{x\to 0}\left(\frac{8-8\mathrm{e}^{-5x}+2x}{\cos x+x-1}\right).$$

a. 42

b. 0

c. ∞

d. $-\infty$

5 Given

$$f(x) = x^2 - 3x,$$

use the definition of the derivative as a limit to calculate $f'(1)$.

a. $\lim_{h \to 0} (h - 1) = -1$

b. $\lim_{h \to 0} (h^2 - 3h - 1) = -1$

c. $\lim_{h \to 0} (2h - 1) = -1$

d. $\lim_{h \to 0} \left(\frac{2h - 3}{3} \right) = -1$

6 Given

$$y = x^{2/7} - 2\mathrm{e}^x,$$

find the derivative of y with respect to x.

a. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{7}x^{-5/7} - 2\mathrm{e}^x$

b. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{7}x^{9/7} - 2\mathrm{e}^x$

c. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{7}x^{-5/7} - 2x\mathrm{e}^{x-1}$

d. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{7}x^{9/7} - 2x\mathrm{e}^{x-1}$

7 Given

$$xy^3 - 4y^2 = x^2,$$

find the derivative of y with respect to x.

a. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - y^3}{3xy^2 - 8y}$

b. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - y^3}{3xy - 8}$

c. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2 - 8y}$

d. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y - 8}$

8 Given

$$x = t^2 \sin(\mathrm{e}^t),$$

find the derivative of x with respect to t.

a. $\frac{\mathrm{d}x}{\mathrm{d}t} = 2t \sin(\mathrm{e}^t) + t^2\mathrm{e}^t \cos(\mathrm{e}^t)$

b. $\frac{\mathrm{d}x}{\mathrm{d}t} = 2t\mathrm{e}^t \cos(\mathrm{e}^t)$

c. $\frac{\mathrm{d}x}{\mathrm{d}t} = 2t \sin(\mathrm{e}^t) + t^2 \cos(\mathrm{e}^t)$

d. $\frac{\mathrm{d}x}{\mathrm{d}t} = 2t \cos(\mathrm{e}^t)$

9 Given

$$p = \arctan\left(q^2\right) + \cos\left(3q\right) = \tan^{-1}\left(q^2\right) + \cos\left(3q\right),$$

find the derivative of p with respect to q.

a. $\dfrac{2q}{\sqrt{1-q^4}} - 3\sin\left(3q\right)$

b. $\dfrac{2q}{q^4+1} - 3\sin\left(3q\right)$

c. $\dfrac{2q}{q^4+1} + 3\sin\left(3q\right)$

d. $\dfrac{2q}{\sqrt{1-q^4}} + 3\sin\left(3q\right)$

10 Given

$$f(x) = \frac{x+1}{x-4},$$

find f'.

a. $f'(x) = -\dfrac{5}{\left(x+1\right)^2}$

b. $f'(x) = \dfrac{5}{\left(x-4\right)^2}$

c. $f'(x) = -\dfrac{5}{\left(x-4\right)^2}$

d. $f'(x) = \dfrac{5}{\left(x+1\right)^2}$

11 Given

$$g(x) = 3\ln\left(x^2+1\right),$$

find g''.

a. $g''(x) = \dfrac{6-6x^2}{\left(x^2+1\right)^2}$

b. $g''(x) = \dfrac{6x}{x^2+1}$

c. $g''(x) = \dfrac{6+18x^2}{\left(x^2+1\right)^2}$

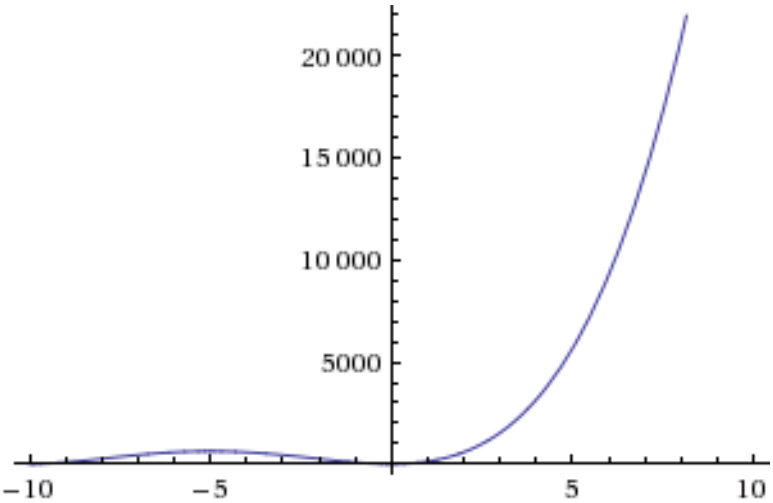
d. $g''(x) = 6x\ln\left(x^2+1\right)$

12 Given

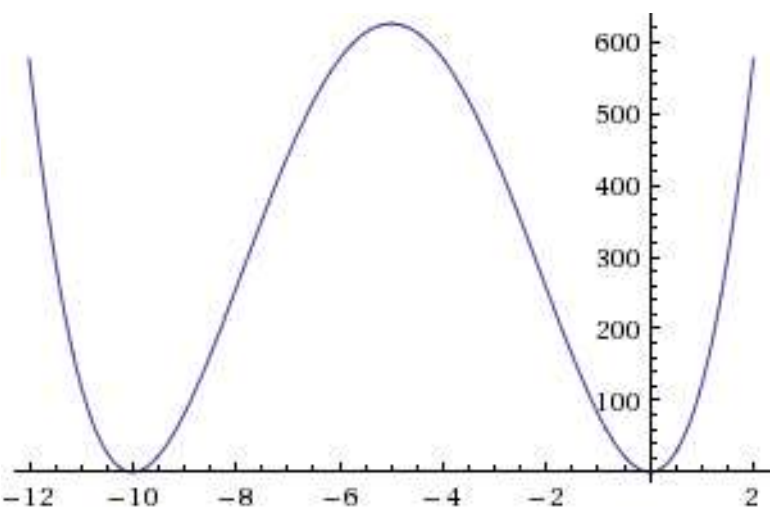
$$f(x) = x^4 + 20x^3 + 100x^2,$$

sketch a graph of f that shows all intercepts (if any), all local extrema (if any), and both infinite limits (if applicable).

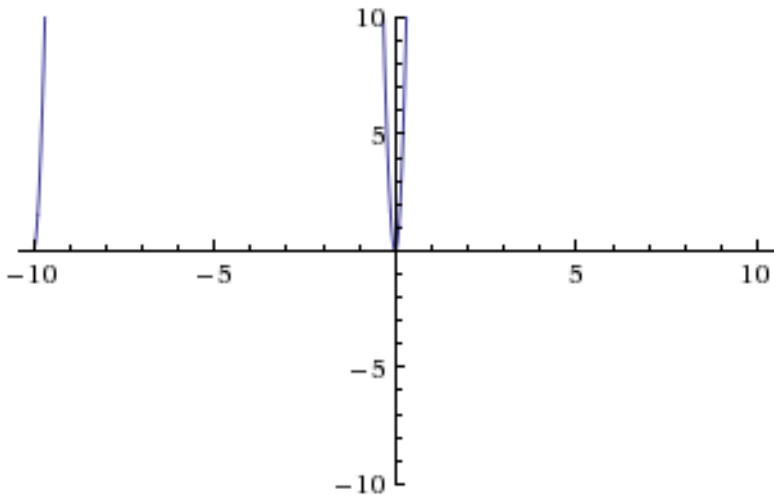
a.

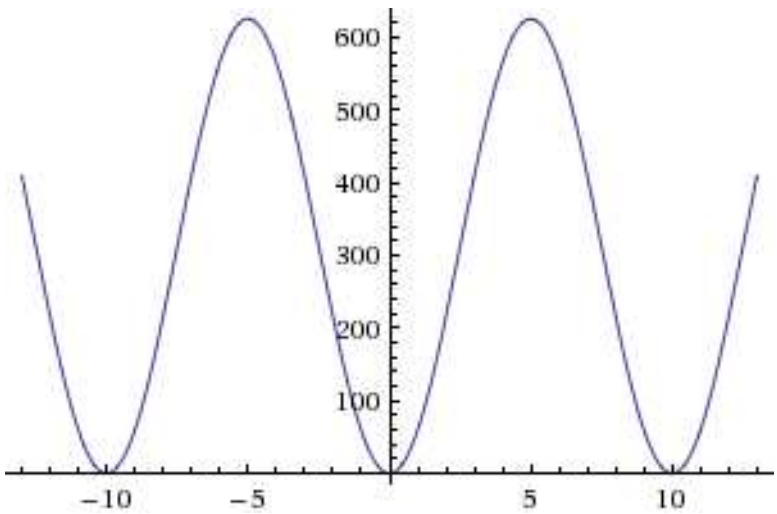


b.



(More answers on the next page)

c. 

d. 

13 Given

$$f(x) = \sqrt{4 - x^2},$$

find the maximum and minimum value of f, if they exist.

a. maximum is 2, minimum is 0

b. maximum is 2, no minimum

c. no maximum, minimum is 0

d. no maximum, no minimum

14 Find the sum

$$\sum_{k=1}^{100} (5k + 3).$$

a. 25,550

b. 25,253

c. 50,503

d. 50,800

15 Given

$$f(x) = 2e^x - \frac{10}{x} + 8x^3,$$

find the antiderivatives (indefinite integrals) of f.

a. $\int f(x)\,dx = 2e^x - 10\ln|x| + 2x^4 + C$

b. $\int f(x)\,dx = 2e^x - \frac{10}{x^2} + 2x^4 + C$

c. $\int f(x)\,dx = 2e^x - 10\ln|x| + 24x^2 + C$

d. $\int f(x)\,dx = 2e^x - \frac{10}{x^2} + 24x^2 + C$

16 Find the value of

$$\int_0^{\pi/3} 4\sec x \tan x\,dx.$$

a. 4

b. 2

c. 0

d. 3

17 Find the value of

$$\int_0^{\pi/2} \frac{\cos x}{\sqrt{4 + 3\sin x}}\,dx.$$

a. $\frac{2}{3}\sqrt{7} - \frac{4}{3} \approx 0.43$

b. 0

c. $2\sqrt{7} - 4 \approx 1.29$

d. $6\sqrt{7} - 12 \approx 3.87$

18 Set up a definite integral whose value is the area bounded by the graphs of these equations:

$$y = x^2,$$
$$y = x + 6.$$

a. $\int_{-2}^{3} (x + 6 - x^2)\,dx$

b. $\int_{-2}^{3} (x^2 - x - 6)\,dx$

c. $\int_{4}^{9} (x + 6 - x^2)\,dx$

d. $\int_{4}^{9} (x^2 - x - 6)\,dx$

19 Set up an integral whose value is the length of the curve with equation

$$y = 2 - \frac{x^2}{4}$$

from $(x, y) = (0, 2)$ to $(x, y) = (2, 1)$.

a. $\int_0^2 \frac{1}{2}\sqrt{4 + x^2}\,\mathrm{d}x$

b. $\int_2^0 \frac{1}{2}\sqrt{4 + x^2}\,\mathrm{d}x$

c. $\int_1^2 \frac{1}{2}\sqrt{4 + x^2}\,\mathrm{d}x$

d. $\int_2^1 \frac{1}{2}\sqrt{4 + x^2}\,\mathrm{d}x$

20 Suppose a block of ice is in the form of a cube and is melting. Suppose further that the length of its sides is decreasing by 1/2 inch per hour. Determine the rate of change in the volume of the block of ice when the length of a side is 12 inches.

a. 216 cubic inches per hour

b. 864 cubic inches per hour

c. 9 cubic inches per hour

d. 15 cubic inches per hour

21 Suppose that a ball thrown into the air has its height given by

$$h = 6 + 5t - 16t^2,$$

where h is its height in feet and t is the time in seconds since it was thrown, from the time it is thrown until the time it lands.

a. How long after it was thrown is it at its maximum height?

b. What is the ball's maximum height?

c. What is the ball's minimum height?

d. At what height was the ball thrown?

e. At what vertical speed was the ball thrown?

f. What is the ball's vertical acceleration?

22 Fill in the blank: If

$$\lim_{x \to c} f(x) = f(c),$$

then f is _____ at c.

23 If

$$\lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

exists, then f is _____ at c.

24 The _____ of y is $\mathrm{d}y$.

25 The _____ of f is f'.

26 The expression $\int_a^b f(x)\,\mathrm{d}x$ is a(n) _____ integral.

27 The expression $\int f(x)\,\mathrm{d}x$ is a(n) _____ integral.

Answers

1 C, 2 B, 3 A, 4 A, 5 A, 6 A, 7 A, 8 A, 9 B, 10 C, 11 A, 12 B, 13 A, 14 A, 15 A, 16 A, 17 A, 18 A, 19 A, 20 A.

21 A $5/32\,\mathrm{s}$, B $409/64\,\mathrm{ft}$, C 0, D $6\,\mathrm{ft}$, E $5\,\mathrm{ft/s}$, F $-32\,\mathrm{ft/s^2}$.

22 continuous

23 differentiable

24 differential

25 derivative

26 definite

27 indefinite