Concavity

There are various terms used when the values of a function, its average rates of change, or its second average rates of change (the rates of change of the rates of change) are all positive (or negative), at least on some interval. When the function is differentiable, and especially when it's twice differentiable, there are easier ways to describe these. This is all summarized in the table below.

Property of f:	Definition:	If differentiable:	If twice differentiable:
Positive	$f(a) > 0$	—	—
Negative	$f(a) < 0$	—	—
Increasing	$\dfrac{f(b)-f(a)}{b-a} > 0$	$f'(a) > 0$	—
Decreasing	$\dfrac{f(b)-f(a)}{b-a} < 0$	$f'(a) < 0$	—
Concave upward	$\dfrac{\dfrac{f(c)-f(b)}{c-b} - \dfrac{f(b)-f(a)}{b-a}}{c-a}$	$\dfrac{f'(b)-f'(a)}{b-a} > 0$	$f''(a) > 0$
Concave downward	$\dfrac{\dfrac{f(c)-f(b)}{c-b} - \dfrac{f(b)-f(a)}{b-a}}{c-a}$	$\dfrac{f'(b)-f'(a)}{b-a} < 0$	$f''(a) < 0$

In all of these, the function f has the given property on some interval if the given condition holds whenever a, b, and c are *distinct* numbers in that interval. (They must be distinct to avoid division by zero.)

Generally, it's much easier to work with the rightmost condition for every property, but you can't do that if the necessary derivatives don't exist. Even if the function isn't differentiable at all, it still makes sense to say whether or not it's concave upward or downward.

Incidentally, here is some other terminology that you may see for these properties:

- Sometimes people use $\geq$ and $\leq$ in place of $>$ and $<$. If you want to be clear, you can use adverbs: 'strictly' for the definitions above (using $>$ and $<$) or 'weakly' for the versions with $\geq$ and $\leq$.
- Sometimes people put the word 'monotone' in front of 'increasing' and 'decreasing', even though it really isn't necessary. (However, when people use this word, they are more likely to mean 'weakly' too, even if they don't say so.)
- Alternatively, if the word 'monotone' is used alone, then it means '(weakly) increasing'; the corresponding word for '(weakly) decreasing' is 'antitone' (but this word is fairly rare).
- If the word 'concave' is used alone, then it means 'concave downard'; the corresponding word for 'concave upward' is 'convex' (and this word is extremely common). Again, people who use this terminology are more likely to mean 'weakly'.